TITLE AND ABSTRACT OF TALKS IN DMHA-18

[Thematic lecture]
Speaker: **Tomasz Przebinda, University of Oklahoma, USA**
Title: What is Howe Correspondence?
Abstract:
Harish-Chandras monumental work was focused on his Plancherel formula for a real reductive group. A complementary approach is that of Roger Howe. He starts with the Weil representation of the metaplectic group. Unlike the left-regular representation, the Weil representation is a minimal representation, almost invisible from the view point of Harish-Chandra. Still, it is a magnificent structure, a sort of Chomolungma in Representation Theory. Howe restricts it to various subgroups, mostly reductive dual pairs, mixes in some ideas from the Classical Invariant Theory and derives far reaching consequences. We shall explain the details.

Speaker: **Amos Nevo, University of Chicago, USA**
Title: Density conjecture and Diophantine approximation
Abstract:
We establish best possible results for exponents in intrinsic quarternion Diophantine approximation.

Speaker: **Angela Pasquale, Université de Lorraine, France**
Title: Geometric analysis on Symplectic spaces and symmetry breaking operators
Abstract:
We consider Howe correspondence for dual pairs with one member compact. Given an irreducible representation Π of the compact member, we study the projection operator P_Π of the Weil representation onto its Π-isotopic component. These operators are examples of symmetry breaking operators in the sense of Kobayashi. Guided by Hermann Weyl's integration formula, we study orbital integrals on the underlying symplectic space. They are needed to compute the Weyl symbols of P_Π.

Our main example will be the dual pair of two compact unitary groups. In this case, we recover by purely analytic methods the existence of Howe correspondence for compact unitary groups, or equivalently, Weyl's First Fundament Theorem for Classical Invariant Theory for complex general linear groups.

This is joint work with Mark McKee and Tomasz Przebinda (University of Oklahoma).

Speaker: **Ankit Bhojak, IISER Bhopal**
Title: Bilinear Bochner-Riesz means for convex domains and Kakeya Maximal function
Abstract:
The study of bilinear Bochner-Riesz means has become an active area of research in harmonic analysis in recent years. The bilinear Bochner-Riesz means of index $\lambda > 0$ associated with the unit disc is the bilinear multiplier operator defined by

$$\mathcal{B}^\lambda(f,g)(x) = \int_{\mathbb{R}^{2n}} (1 - |\xi|^2 - |\eta|^2)_+^\lambda \hat{f}(\xi)\hat{g}(\eta)e^{2\pi i x\cdot(\xi+\eta)}\, d\xi d\eta.$$

The study of L^p-estimates for the operator $\mathcal{B}^\lambda$ was initiated by Bernicot and Germain (2013). Exploiting the symbol $(1 - |\xi|^2 - |\eta|^2)_+^\lambda$ and relying on the results for the linear Bochner-Riesz operators, the range of the boundedness for B^λ were improved by Jeong and Lee (2020) and later, by Kaur and Shrivastava (2022).

In this talk, we generalize the notion of bilinear Bochner-Riesz means in the context of open and bounded convex domains in the plane $\mathbb{R}^2$ and obtain their L^p-bounds. We note that the previous methods employed in the case of unit disc does not extend to the general case. Instead, we rely on the classical approach of using the Kakeya maximal functions to prove L^p-boundedness results for Fourier multipliers to the bilinear setting. In this regard, we introduce the bilinear Kakeya maximal function in the plane and study its L^p-boundedness properties.
(This is a joint work with Surjeet Singh Choudhary and Saurabh Shrivastava.)

Speaker: **Arun Kumar Bhardwaj, IIT Guwahati**
Title: Hilbert Transform, Nevanlinna Class and Toeplitz kernels
Abstract:
The celebrated integral transforms such as Fourier transform, Laplace transform, and Hilbert transform have tremendous applications in various branches of science and engineering. However, unlike to Fourier or Laplace transform, very few functions have an explicit formula for their Hilbert transforms. In this article we obtain an explicit formula for the Hilbert transform of $\log|f|$, for the function$f$ in Nevanlinna class having continuous extension to the real line. This family is the largest possible for which such a formula for the Hilbert transform of $\log|f|$, can be obtained. The formula is very general and implies several previously known results.

Speaker: **Baklouti Ali, Faculty of Sciences, University of Sfax, Tunisia**
Title: The polynomial Conjecture for monomial representations of discrete type of an exponential solvable Lie group
Abstract:
Aligning with Harish-Chandra's pioneer works on the representation theory of reductive Lie groups, I will define the notion of a monomial representation $\tau = \mathrm{ind}_H^G\chi$ of discrete type, where $G = \exp\mathfrak{g}$ is an exponential solvable Lie group, H an analytic subgroup of G, and χ a unitary character of H. I will then talk about the related Plancherel theory and consider the polynomial conjecture, stating that the algebra $D_\tau(G/H)$ of G-invariant differential operators over the line bundle associated with the data (G, H, χ) consists of a polynomial ring, isomorphic to the algebra $\mathbb{C}[\Gamma_\tau]^H$ of H-invariant polynomial functions on an affine subspace $\Gamma_\tau \subset \mathfrak{g}^*$.

This is a joint work with H. Fujiwara and J. Ludwig.

Speaker: **Bernhard Johann Krötz, Universität Paderborn, Germany**
Title: Norms on Harish-Chandra modules
Abstract:
I will report on work in progress with Bernstein, Ganguly, Kuit and Sayag. It is about to compare G-continuous norms on a Harish Chandra module with a fixed growth rate. We show that there is a maximal and minimal equivalence class of such norms and measure their Sobolev distance. This yields a new invariant which we call Sobolev gap. We provide upper and lower bounds for the Sobolev gap of a Harish Chandra module for $Sl(2,\mathbb{R})$.

Speaker: **Chandan Biswas, IIT Bombay**
Title: On extremizers for a certain restriction operator
Abstract:
We will discuss the question of existence of extremizers for Fourier restriction onto the moment curve.

Speaker: **Erik van den Ban, Utrecht University, Netherlands**
Title: Harish-Chandra's philosophy of cuspforms for Whittaker functions.
Abstract:
In 1982 Harish-Chandra announced to have obtained a proof of the Plancherel theorem in the context of Whittaker functions on a real reductive group. Furthermore, he gave a complete formulation of the theorem.

Because of his untimely death in 1983, this work was only published in 2018, as a paper in the posthumous volume 5 of his collected works, edited by V.S. Varadarajan and R. Gangoll. In the paper Harish-Chandra explains the cusp form philosophy for Whittaker functions. In the talk I will explain this viewpoint, and will report on recent results of myself that allow a complete proof of the Whittaker Plancherel decomposition.

Speaker: **Eyal Subag, Bar-Ilan University, Israel**
Title: Families of Harish-Chandra modules and the hydrogen atom
Abstract:
In this talk I shall explain how the Schrödinger equation of the hydrogen atom gives rise to an algebraic family of Harish-Chandra pairs that is related to $SO(4)$, $SO(3,1)$ and the corresponding Cartan motion group. I will show that solutions of the Schrödinger equation form an algebraic family of Harish-Chandra modules. If time permits, I will explain how the infinitesimally unitary Jantzen quotients of the family of solutions are related to spectral decomposition of the Schrödinger operator.

Speaker: **Haripada Roy, IIT Kanpur**
Title: Fractional Caffarelli-Kohn-Nirenberg type inequalities on the Heisenberg group
Abstract:
In 1984, L. Caffarelli and his coauthors established a family of interpolation inequalities, now known as Caffarelli-Kohn-Nirenberg (CKN) inequalities, which includes the well known Sobolev inequality and Hardys inequality as particular cases. In this talk first we will briefly discuss the classical and fractional Sobolev, Hardys and CKN inequalities for Euclidean spaces. Then we will discuss our recent results concerning fractional CKN type inequalities on the Heisenberg group, which includes the fractional Sobolev and Hardy type inequalities established by Adimurthi and Arka Mallick. Our inequalities also give an improvement on the range of indices for the Hardy type inequality established by them.

Speaker: **Jan Frahm, Aarhus University, Denmark**
Title: Minimal representations of exceptional groups and theta correspondence
Abstract:
The classical theta correspondence establishes a bijection between certain sets of representations of two groups that occur as a dual pair inside a symplectic group. Roughly speaking, two representations are in correspondence if their tensor product, viewed as a representation of the product group, occurs in the restriction of the metaplectic representation of the symplectic group (or rather its double cover) to the product group. This correspondence has proven to be a strong tool not only in representation theory, but also in the construction of automorphic forms or in classical harmonic analysis.

Despite its importance, one of the big disadvantages of the classical theta correspondence is that only classical groups occur as members of dual pair inside a symplectic group. It is therefore desirable to extend the correspondence to dual pairs in more general reductive groups, replacing the metaplectic representation by a so-called minimal representation of this group. I will explain how to explicitly construct some of these minimal representations and how to use these explicit models to obtain theta correspondences for dual pairs where one member is $SL(2,\mathbb{R})$ and the other member is a non-compact exceptional group.

Speaker: **Job Jacob Kuit, Paderborn University, Germany**
Title: The most continuous part of the Plancherel decomposition for a real spherical space
Abstract:
Let Z be a homogeneous space of a real reductive group G. The Plancherel decomposition of Z is the decomposition of the space $L^2(Z)$ of square integrable functions into a direct integral of irreducible unitary representations of G. In general this decomposition has a mixed discrete and continuous nature. The closed G-invariant subspace of $L^2(Z)$ that decomposes into the largest continuous families is called the most continuous part. In this talk I will report on joint work with Eitan Sayag in which we determine the Plancherel decomposition of the most continuous part of $L^2(Z)$ for real spherical homogeneous spaces Z.

Speaker: **Manish Chaurasia, IIT(BHU) Varanasi**
Title: A Generalization of a result of Vemuri
Abstract:
Assuming that a function and its Fourier transform are dominated by Gaussians, a sharp estimate for the rate of exponential decay of its Hermite coefficients is obtained in terms of the variances of the dominating Gaussians.

Speaker: **Michael Cowling, University of New South Wales, Australia**
Title: Harish-Chandra's asymptotic expansion and decomposition of representations
Abstract:
Irreducible unitary representations of semisimple Lie groups, and other admissible representations, have K-finite matrix coefficients that admit asymptotic expansions at infinity, and in the higher rank case, there are different ways of approaching infinity and different expansions for each of these ways.

We show that the expansion as one approaches infinity "away from the walls of the Weyl chamber" controls the behaviour of all matrix coefficients in all directions.

This may be used to control the representations that appear in the decomposition of a general unitary representation in a much more precise way than was previously known.

Speaker: **Nobukazu Shimeno, Kwansei Gakuin University, Japan**
Title: Matrix-valued spherical functions on semisimple Lie groups
Abstract:
Harish-Chandra's c-function on a real semisimple Lie group gives the leading coefficient of the zonal spherical function and determines the Plancherel measure for the spherical transform. Gindikin and Karpelevič gave an explicit formula for the c-function. Moreover, Heckman and Opdam developed a theory of hypergeometric functions associated with root systems, which are generalizations of zonal spherical functions.

In the case of spherical functions for non-trivial K-types, explicit formulae for c-functions and spherical inversions have been known for a few cases, including the case of one-dimensional K-types. In this talk, I will explain that for certain class of K-types, associated elementary spherical functions can be written by Opdam's non-symmetric hypergeometric functions. As corolaries, we have explicit formulae of c-functions and inversion formulae for the spherical transforms.

This talk is based on joint work with Hiroshi Oda.

Speaker: **Rabeetha V, IIT Madras**
Title: Twisted Shift-Invariant system in $L^2(\mathbb{R}^{2n})$
Abstract:
We consider a general twisted shift-invariant system, $V^t(A)$ on $\mathbb{R}^{2n}$, consisting of twisted translates of countably many generators and study the problem of obtaining a characterization for the system $V^t(A)$ to form a frame sequence or a Riesz sequence. We illustrate our theory with some examples. We also obtain an orthonormal sequence of twisted translates from a given Riesz sequence of twisted translates. This is a joint work with Santi Ranjan Das and Radha Ramakrishnan which is appeared in Nagoya Mathematical Journal (2023).

Speaker: **Ramesh Gangolli, University of Washington, USA**
Title: TBA
Abstract:
TBA

Speaker: **Radouan Daher, University Hassan Morocco, Morocco**
Title: Discrete Fourier Jacobi transform and generalized Lipschitz classes
Abstract:
In this talk, we use the methods of Fourier-Jacobi harmonic analysis to generalize Boas-type results. We give necessary and sufficient conditions in terms of the Fourier-Jacobi coefficients of a function f in order to ensure that it belongs either to one of the generalized Lipschitz classes.

Speaker: **Riju Basak, IISER Mohali**
Title: On sharp estimate for the solution of the wave equation associated with the twisted Laplacian
Abstract:
The sharp fixed-time Lebesgue and Hardy space estimates for the solution of the Cauchy problem associated with the standard Euclidean Laplacian were first studied independently by A. Miyachi and J. Perel in 1980. After that, many mathematicians studied this problem for various classes of operators on different settings but the sharp estimate is still not available for many operators.

In this talk, we shall discuss fixed-time estimates for the solution of the Cauchy problem associated with the twisted Laplacian. This talk is based on a joint work with Jotsaroop Kaur.

Speaker: **Ritika Singhal, IIT Delhi**
Title: Paley inequality for the Weyl transform and its applications
Abstract:
In this talk, we prove several versions of the classical Paley inequality for the Weyl transform. As an application, we discuss L^p-L^q boundedness of the Weyl multipliers and prove a version of the Hörmander's multiplier theorem. We also prove Hardy-Littlewood inequality. Finally, we study vector-valued versions of these inequalities. In particular, we consider the inequalities of Paley, Hausdorff-Young, and Hardy-Littlewood and their relations.

Speaker: **Sanjay Parui, NISER Bhubaneswar**
Title: Hardy-Littlewood-Sobolev Inequality for Upper Half Space
Abstract:
We define an extension operator and study (L^p, L^q) boundedness of Hardy-Littlewood-Sobolev inequality and weighted Hardy-Littlewood-Sobolev inequality on upper Half space for the Dunkl transform.

Speaker: **Santosh Nayak, NISER Bhubaneswar**
Title: Weyl transform on some nonunimodular groups
Abstract:
For $p > 2$, B. Simon studied the unboundedness of the Weyl transform for symbol belonging to $L^p(\mathbb{R}^n \times \mathbb{R}^n)$. In this talk, we study the analog of unboundedness of the Weyl transform on some nonunimodular groups, namely, the affine group, similitude group, and affine Poincaré group.

Speaker: **Shubham R. Bais, IIT Hyderabad**
Title: Boundedness of a class of integral operators on the Bergman space over the Upper Half-Plane
Abstract:
Let Π denote the upper half-plane and $\mathcal{A}^2(\Pi)$ be the Bergman space over the upper half-plane. In this talk, we define a class of integral operators on the space $\mathcal{A}^2(\Pi)$. We characterize the integral kernels so that the operators are bounded. We also discuss various operator theoretic properties and a C^*-subalgebra of it which is generated by Toeplitz operators with special symbols. This is based on the joint work with D. Venku Naidu and Pinlodi Mohan.

Speaker: **Siddhartha Sahi, Rutgers University, New Brunswick NJ, USA**
Title: Lyapunov exponents for random products of real matrices
Abstract:
We consider probability measures on $GL(n,\mathbb{R})$ that are left-invariant under the orthogonal group $O(n,\mathbb{R})$. For any such measure we consider the following two quantities: (a) the mean of the log of the absolute value of the eigenvalues of the matrices and (b) the Lyapunov exponents of random products of matrices independently drawn with respect to the measure. Our main result is a lower bound for (a) in terms of (b).

This lower bound was conjectured by Burns-Pugh-Shub-Wilkinson (2001), and special cases were proved by Dedieu-Shub (2002), Avila-Bochi (2003) and Rivin (2005). We give a proof in complete generality by using some results from the theory of spherical functions and Jack polynomials.

This is joint work with Diego Armentano, Gautam Chinta, and Michael Shub. ($arXiv$: 2206.01091), (Ergodic theory and Dynamical systems, to appear).

Speaker: **Simon Marshall, University of Wisconsin, USA**
Title: Bounds for eigenfunctions on locally symmetric spaces in the large eigenvalue limit
Abstract:
Let X be a locally symmetric space, and f a joint eigenfunction of the invariant differential operators on X. I will discuss the question of how much f can concentrate on small sets as its spectral parameter goes to infinity. This concentration will be measured in terms of the L^p norms of f, and its integrals and L^p norms along submanifolds. I will survey a range of results, proved using tools from both harmonic analysis and number theory.

Speaker: **Somnath Ghosh, IISc Bangalore**
Title: Dynamical Uncertanity principle on the Heisenberg group
Abstract:
The Uncertainty Principle for a function f and its Fourier transform $\hat{f}$ is a central phenomena in Euclidean harmonic analysis. Consider the Schrödinger equation on $\mathbb{R}^n$:

$$i\partial_t u(x,t) + \Delta_x u(x,t) = 0, \quad u(x,0) = u_0(x).$$

Then any Uncertainty Principle for the pair $(f, \widehat{f})$ can be restated in terms of an Uncertainty Principle for (u_0, u_1), known as a Dynamical Uncertainty Principle, where $u_1(x) = u(x,1)$.

In this talk, we will deal with the Schrödinger equation associated to the sub-Laplacian on the Heisenberg group. Besides considering certain Uncertainty Principle for solutions of the free Schrödinger equation on the Heisenberg group, we will see the influence of the potential. Further, we will discuss some limitations to Dynamical Uncertainty Principles. (Joint work with Prof. Philippe Jaming).

Speaker: **Suman Mukherjee, NISER Bhubaneswar**
Title: Weighted Bilinear Multiplier Theorems in Dunkl Setting via Singular Integrals
Abstract:
Dunkl theory is a generalization of Fourier analysis and special function theory related to root systems and reflection groups. The Dunkl operators, introduced by Charles Dunkl, can be considered as generalizations of ordinary directional derivatives. Through the well-established connection between the Fourier transform and the partial derivative operator, Dunkl operators introduce a new operator that generalizes the classical Fourier transform, called the Dunkl transform. This signifies the commencement of the analytical aspect of Dunkl theory, a thorough initiative to generalize the results of classical Fourier analysis and the theory of special functions within the framework of root systems and reflection groups. The goal of this talk is to present weighted inequalities for bilinear multiplier operators in the Dunkl setting,with multiple Muckenhoupt weights,using the theory of multilinear Calderón-Zygmund type operators in the Dunkl set up. This is based on a joint work with Sanjay Parui.

Speaker: **Surjeet Singh Choudhary, IISER Bhopal**
Title: Endpoint estimates for the Bilinear Spherical Maximal function
Abstract:
The study of maximal averaging operators plays a fundamental role in analysis. One such operator is the spherical maximal function, defned as

$$A_*f(x) = \sup_{t>0} \Big| \int_{\mathbb{S}^{d-1}} f(x-ty)d\sigma(y) \Big|.$$

The L^p-boundedness of A_* was proved by Stein (1976) for $p > \frac{d}{d-1}, d \geq 3$ and Bourgain (1986) in dimension $d = 2$ for $p > 2$.

In this talk, we consider the bilinear analogue of the spherical maximal function given by

$$M(f,g)(x) = \sup_{t>0} \Big| \int_{\mathbb{S}^{2d-1}} f(x-ty)g(x-tz)d\sigma(y,z) \Big|.$$

In dimension $d \geq 2$, Lee and Jeong (2020) proved sharp $L^p(\mathbb{R}^d) \times L^q(\mathbb{R}^d) \to L^r(\mathbb{R}^d)$ bounds for bilinear spherical maximal function when $r > \frac{d}{2d-1}$. For $r = \frac{d}{2d-1}$, they obtained restricted weak type bounds in dimension $d \geq 3$. Dosidis and Ramos (2022) proved sharp bounds in dimension $d = 1$ for $p, q > 2$ and $\frac{1}{r} = \frac{1}{p} + \frac{1}{q}$. We prove restricted weak type estimate at the endpoints for bilinear spherical maximal function in dimensions $d = 1, 2$. We also prove sharp L^p improving bounds for localised bilinear spherical maximal function.

This is a joint work with Ankit Bhojak, Saurabh Shrivastava and Kalachand Shuin.

Speaker: **Tapendu Rana, Ghent University, Belgium**
Title: Weighted estimates for Hardy-Littlewood Maximal functions on Harmonic N A group
Abstract:
For a locally integrable function f on $\mathbb{R}^d$, its Hardy-Littlewood maximal function $M_{\mathbb{R}^d} f$ is defined as

$$M_{\mathbb{R}^d} f = \sup_{r>0} \frac{1}{|B(r,x)|} \int_{B(r,x)} |f(y)|\, dy$$

It is well-known that the operator is bounded on L^p for $p > 1$ and of weak type $(1,1)$. In the seminal works of Fefferman-Stein and Muckenhoupt, they provided a characterization of weights on $\mathbb{R}^d$ by introducing the classical A_p weight condition, which allows the weighted boundedness of the Hardy-Littlewood maximal operator.

In this talk, we will discuss the weighted boundedness of the Hardy-Littlewood maximal operator in Harmonic N A groups, also known as Damek-Ricci spaces. More precisely, we will demonstrate that the Euclidean type A_p condition is not necessary for the Hardy-Littlewood maximal operator to be bounded in this setting, making it apparent that such conditions are not suitable in Harmonic N A groups. We provide a necessary condition and define a suitable notion of admissible A_p class of weights for which the maximal operator is weighted bounded. Furthermore, as an endpoint case, we will prove a variant of the Fefferman-Stein inequality.

This talk is based on a joint work with Pritam Ganguly and Jayanta Sarkar.

Speaker: **Toshio Oshima, Josai University, Japan**
Title: Generalized hypergeometric functions with several variables
Abstract:
A restriction of the zonal spherical function of type A on a singular line is expressed by the generalized hypergeometric function (HG)

$${}_pF_{p-1}(\alpha_1, \dots, \alpha_p; \alpha'_1, \dots, \alpha'_{p-1}; x) = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \cdots (\alpha_p)}{(\alpha'_1)_n \cdots (\alpha'_{p-1})_n} \frac{x^n}{n!}.$$

Then the Gauss summation formula, or Harish-Chandra's c-function, of Heckman-Opdam's HG, which is a generalization of the zonal spherical function, is obtained by a connection formula of this generalized HG by [1].

We introduce the generalized HG of two variables

$$\phi(x,y) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(\alpha_1)_m \cdots (\alpha_p)_m (\beta_1) \cdots (\beta_q)_n (\gamma_1) \cdots (\gamma_r)_{m+n}}{(\alpha'_1)_m \cdots (\alpha'_{p'})_m (\beta'_1) \cdots (\beta'_{q'})_n (\gamma'_1)_{m+n} \cdots (\gamma'_{r'})_{m+n}} \frac{x^m y^n}{m! n!}$$

under the condition that the differential equation $\mathcal{M}$ satisfied by $\phi(x,y)$ has no irregular singularities, namely we assme

$$p' - p + 1 = q' - q + 1 = r - r'.$$

We note that Appell's hypergeometric functions are examples.

The following problems will be discussed.

- Integral repsentaion of $\phi(x,y)$
- Rank and singularities of $\mathcal{M}$
- Construction of a base of local solutions at several singular points
- Connection fromula between these local solutions
- Necessay and sufficient condition for the irreducibility of $\mathcal{M}$
- Genelization of $\phi(x,y)$ to HG with more variables.

This work is in progress collabolated with S-J. Matsubara-Heo.

References

[1] T. Oshima and N. Shimeno, Heckman-Opdam hypergeometric functions and their specializations, RIMS Kôkyûroku Bessatsu **B20** (2010), 129–162.

[2] T. Oshima, Integral transformations of hypergeometric functions with several variables,

Speaker: **Toshiyuki Kobayashi, The University of Tokyo, Japan**

Title: Harish-Chandras admissibility theorem and beyond

Abstract:

Let G be a real reductive linear Lie group, and K a maximal compact subgroup of G. Harish-Chandra's admissibility theorem asserts that any irreducible unitary representation decomposes into a direct sum of irreducible K-modules with each multiplicity finite. Such a theorem does not hold if we replace the Riemannian symmetric pair (G, K) by a reductive symmetric pair (G, G') in general. We explore a "nice" framework for the restriction of an irreducible representation of G to the subgroup G' in this generality with focus on finite/uniformly bounded multiplicity property.

If time permits, I also will discuss its application to analysis of locally pseudo-Riemannian symmetric spaces.

Speaker: **Utsav Dewan, ISI Kolkata**

Title: Restricted Mean Value Property on Riemannian Manifolds

Abstract:

A continuous function in an Euclidean domain is harmonic if and only if it satisfies the spherical mean value property for all spheres contained in that domain. But what happens if a continuous function satisfies instead the following 'restricted mean value property': for each point in the domain it satisfies the mean value property precisely on one such sphere (centered at the point). Then is the function still going to be harmonic? This is the classical 'one-circle problem' posed by Littlewood. We will see some results dealing with sufficient conditions in terms of the boundary behavior of the function for the above problem to have an affirmative answer in the setting of (1) domains in Riemannian manifolds and (2) Hadamard manifolds of pinched negative sectional curvature, extending classical results of Fenton for the Euclidean unit disc. This is based on a joint work with Prof. Kingshook Biswas.