RANDIĆ INDEX OF A GRAPH WITH SELF-LOOPS

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Let $G(n, m)$ be simple a graph with vertex set V and $S \subseteq V$ with $|S| = \sigma$. The graph G_S is obtained by adding a self-loop to each vertex of the graph G in the set S. The Randić index of a graph is one of the important degree based topological indices which has its application in chemistry. In this manuscript, the Randić index of a graph with self-loop is defined and obtained bounds for Randić index of regular graph, tree and complete bipartite graph with self-loops.

ECCENTRIC GRAPH OF TREES AND THEIR CARTESIAN PRODUCTS

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Let G be an undirected simple connected graph. We say a vertex u is eccentric to a vertex v in G if $d(u,v) = \max\{d(v,w) : w \in V(G)\}$. The eccentric graph, $E(G)$ of G is a graph defined on the same vertex set as of G and two vertices are adjacent if one is eccentric to the other. We find the structure and the girth of the eccentric graph of trees and see that the girth of the eccentric graph of a tree can either be zero, three, or four. Further, the eccentric girth of the Cartesian product of trees can either be zero, three, four or six. We provide a comprehensive classification when the eccentric girth assumes these values. We also give the structure of the eccentric graph of the grid graphs and the Cartesian product of cycles. Finally, we determine the conditions under which the eccentricity matrix of the Cartesian product of trees becomes invertible.

ON SOME GENERAL OPERATORS OF HYPERGRAPHS

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Here we introduce connectivity operators, namely, diffusion operators, general Laplacian operators, and general adjacency operators for hypergraphs. These operators are generalizations of some conventional notions of apparently different connectivity matrices associated with hypergraphs. In fact, we introduce here a unified framework for studying different variations of the connectivity operators associated with hypergraphs at the same time. Eigenvalues and corresponding eigenspaces of the general connectivity operators associated with some classes of hypergraphs are computed. Applications such as random walks on hypergraphs, dynamical networks, and disease transmission on hypergraphs are studied from the perspective of our newly introduced operators. We also derive spectral bounds for the weak connectivity number, degree of vertices, maximum cut, bipartition width, and isoperimetric constant of hypergraphs.

ALTERNATING SIGN MATRICES WITH VERTICAL SYMMETRY AND PLANE PARTITIONS

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Alternating sign matrices (ASMs) are square matrices with entries in the set $\{0, 1, -1\}$, where the non-zero entries alternate in sign along rows and columns, with all row and column sums being 1. Plane partitions are 3-D generalizations of ordinary integer partitions. We discuss some new connections between these two classes of objects thereby connecting an already studied class of plane partitions with a new class of ASMs related to vertically symmetric ASMs. We also give the most general enumeration result for several symmetry classes of ASMs generalising several known results.

HYPERGEOMETRIC FUNCTIONS FOR DIRICHLET CHARACTERS AND PEISERT-LIKE GRAPHS ON $\mathbb{Z}_N$

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For a prime $p \equiv 3 \pmod 4$ and a positive integer t, let $q = p^{2t}$. The Peisert graph of order q is the graph with vertex set $\mathbb{F}_q$ such that ab is an edge if $a-b \in \langle g^4 \rangle \cup g\langle g^4 \rangle$, where g is a primitive element of $\mathbb{F}_q$. In this paper, we construct a similar graph with vertex set as the commutative ring $\mathbb{Z}_n$ for suitable n, which we call *Peisert-like* graph and denote by $G^*(n)$. Owing to the need for cyclicity of the group of units of $\mathbb{Z}_n$, we consider $n = p^\alpha$ or $2p^\alpha$, where $p \equiv 1 \pmod 8$ is a prime and α is a positive integer. For primes $p \equiv 1 \pmod 8$, we compute the number of cliques of orders three and four in the graph $G^*(p^\alpha)$ by evaluating certain character sums. To find the number of cliques of order four, we first introduce hypergeometric functions containing Dirichlet characters as arguments, and then express the number of cliques of order four in $G^*(p^\alpha)$ in terms of these hypergeometric functions.

ON GENERALIZED RANDIĆ MATRIX OF DIGRAPHS

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For a directed graph S with Randić matrix $R(S) = (r_{ij})$, where $r_{ij} = 1/\sqrt{d_i^+ d_j^-}$ if (v_i, v_j) is an arc of S and 0 otherwise. For the outdegrees diagonal matrix, $D(S)$ of S we can study the convex combination of the matrices $R(S)$ and $D(S)$. Keeping in this mind, we propose a generalized Randić matrix as $R_\alpha(S) = \alpha D(S) + (1-\alpha)R(S)$ for $0 \leq \alpha \leq 1$. The eigenvalue of $R_\alpha(S)$ with the largest modulus is called the $R_\alpha(S)$-spectral radius of S. In this paper, we obtain some bounds for the $R_\alpha(S)$-spectral radius of this newly defined matrix and extremal graphs attained these bounds are characterized. Finally, we find some bounds of the generalized Randić energy for digraphs.

Keywords: Directed Graphs, Generalized Randić matrix, Generalized Randić energy

AMS Classifications (2010): 05C50; 05C20; 05C31

ON ZAGREB ENERGIES OF SOME GRAPH OPERATIONS

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Recently, Zagreb energies, a graph invariants based on the eigenvalues of the Zagreb matrices have been proposed as an analogous to graph energy. In this communication, the Zagreb energies and Zagreb spectral radius are examined in relation to a number of graph operations, such as m-splitting graphs, m-shadow graphs, m-duplicate graphs, and extended bipartite double graphs. Further, we explore these generalised graphs within the context of specific graph types such as complete graphs, complete bipartite graphs, cycle graphs, and the complements of cycle graphs. Furthermore, we report an error presented by Sheikholeslami et al. (2021) that contradicts the claim of hyperenergetic behaviour for the splitting graph of regular graphs and also we establish the non-hyperenergetic behaviour of the m-shadow graph.

Keywords: Graph energy, Zagreb energy, Graph operations.
AMS Classifications (2010): 05C50, 05C76.

SOME RESULTS ON A_α- SPECTRA OF GRAPHS WITH POCKETS

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Let F and H_v be two simple connected graphs and let v be a specified vertex of H_v and $u_1, u_2, \cdots, u_k \in F$. Then the graph $G = G[F, u_1, u_2, \cdots, u_k, H_v]$ obtained by taking one copy of F and k copies of H_v, and then attaching the ith copy of H_v to the vertex u_i, $i \in \{1, 2, \cdots, k\}$ (identify u_i with the vertex v of the ith copy) is called a graph with k pockets. The copies of the graph H_v that are attached to the vertices u_i, $i \in \{1, 2, \cdots, k\}$ are called pockets. In this communication, we present A_α-eigenvalues and corresponding A_α-eigenvectors of graphs with k- pockets. In addition, we present A_α- spectra with respect to some unary and binary operations on graphs as well as for generalised star graph.

Keywords: Graph with k pockets, A_α-spectra, A_α-eigenvectors, Generalised star, graph operations.

ECCENTRICITY MATRIX OF DISTANCE REGULAR GRAPHS

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A connected graph with diameter d is said to be distance regular if for any two vertices, say x and y which are at distance i then there exist c_i number of vertices at distance $i-1$, a_i vertices at distance i and b_i vertices at distance $i+1$ from x among the neighbours of the vertex y.

The eccentricity of a vertex $u \in G$ is defined as $e(u) = max\{d(u,v) : v \in V(G)\}$, where $d(u,v)$ is the distance between the vertices u and v. The eccentricity matrix $\varepsilon(G)$ is derived from the distance matrix, whose $(u,v)^{th}$ element of the matrix is $d(u,v)$ if $d(u,v) = \min\{e(u), e(v)\}$, otherwise 0.

We compute the eccentricity spectrum and spectral radius of some well-known families of distance regular graphs. Further, we find conditions for when the eccentricity matrix of these graphs will be irreducible.

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STRUCTURAL CHARACTERIZATION OF CONNECTED GRAPHS WITH INTEGER-VALUED Q-SPECTRAL RADIUS

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The Q-eigenvalues are the eigenvalues of the signless Laplacian $Q(G)$ of a graph G and the largest Q-eigenvalue is known as the Q-spectral radius $q(G)$ of G. The edge-degree of an edge is defined as the number of edges adjacent to it. In this article, we characterize the structure of simple connected graphs having integral Q-spectral radius. We show that the necessary and sufficient condition for such graphs to contain either a double star $\mathcal{S}_r^2$ or its variation $\mathcal{S}_r^{2,1}$ (having exactly one common neighbor between the central vertices) as a subgraph is that the maximum edge-degree is $2r$, where $r = q(G) - 3$. In particular, we characterize all graphs that contain only double star as a subgraph when $q(G)$ equals 8 and 9. Further, we characterize all the connected edge-non-regular graphs having maximum edge-degree equal to 4 whose minimum and maximum Q-eigenvalues are integers.

ON THE FIRST ZAGREB INDEX OF GRAPHS WITH SELF-LOOPS

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Some of the most comprehensively studied degree-based topological indices are the Zagreb indices. The first Zagreb index $M_1(G)$ of a graph G is defined as the sum of squares of the degrees of the vertices. Let $X \subseteq V(G)$ and let G_X be the graph obtained from the simple graph G, by attaching a self-loop to each of its vertices belonging to X. In this article, some sharp upper and lower bounds for the first Zagreb index of graphs with self-loops using the total number vertices, edges and self-loops has obtained. Also, an attempt to bound $M_1(G_X)$, using energy and spectral radius of the adjacency matrix of the graph has been put forward. Finally, a Nordhaus-Gaddum type of upper and lower bound for the $M_1(G_X)$ has been obtained.

ALMOST SELF-CENTERED INDEX OF SOME GRAPHS

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For a simple connected graph G, center $C(G)$ and periphery $P(G)$ are subgraphs induced on vertices of G with minimum and maximum eccentricity, respectively. An n-vertex graph G is said to be an almost self-centered (ASC) graph if it contains $n-2$ central vertices and two peripheral (diametral) vertices. An ASC graph with radius r is known as an r-ASC graph. The r-ASC index of any graph G is defined as the minimum number of new vertices, and required edges, to be introduced to G such that the resulting graph is r-ASC graph in which G is induced. For $r = 2$ and 3, r-ASC index of a few graphs is calculated by [1] and [2], respectively. In this paper, we give bounds to the r-ASC index of diameter two graphs. We also construct graphs satisfying upper and lower bounds of r-ASC index of graphs with diameter two. Finally, we determine the exact value of r-ASC index of cycles and paths for $r \geq 4$.

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EXTRACTION AND COMBINATION METHOD OF BOOLEAN LOGIC SIMPLIFICATION

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The minimization or simplification of Boolean algebraic expressions or logic gates is vital before their implementation in hardware. A reduced number of gates not only decreases hardware costs but also mitigates heat generation and significantly enhances processing speed. This article introduces the *"Extraction and Combination Method,"* a novel and straightforward approach for minimizing Boolean expressions. Leveraging the power of Boolean algebraic axioms, the method involves systematic step-by-step algebraic manipulation of Boolean expressions through a sequence of extractions and combinations of subexpressions within the given Boolean function. This process simplifies the expressions to their simplest forms. Unlike complex algorithms such as Espresso and others that demand high technical proficiency, this new simplification algorithm is beginner-friendly and easily understandable. Despite its simplicity, the method is potentially powerful as it can handle any number of terms and variables. The article presents the algorithm, outlines the step-by-step procedure, and provides illustrative examples to elucidate the method's application. Additionally, it offers a brief comparison with existing methods such as Karnaugh Maps, Quine-McCluskey Algorithm, Classical Algebraic Method, Espresso Algorithm, and the Truth Graph method. The article concludes by emphasizing the uniqueness of the simplified form achieved through this method and asserts the termination of the algorithmic process.[1]

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KG SOMBOR ENERGY OF GRAPHS WITH SELF LOOPS

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Let $G = (V, E)$ be a connected graph. A topological invariant named KG Sombor index was introduced by V. R. Kulli, defined as $KG = KG(G) = \sum\limits_{ue} \sqrt{d(u)^2 + d(e)^2}$, where $\sum\limits_{ue}$ indicates summation over vertices $u \in V(G)$ and the edges $e \in E(G)$ that are incident to u. In this paper, we extend the KG Sombor index of simple graphs to graph with self loops. We study some properties of KG Sombor eigenvalues and few bounds for KG Sombor energy of graphs with self loops and KG Sombor characteristic polynomial of graphs with self loops. Also, we find that $EKG(G_S)$ has an excellent correlation with total π-electron energies of hetero-atoms, with correlation coefficient 0.990.