GENERALIZED BOHR INEQUALITIES FOR CERTAIN CLASSES OF FUNCTIONS ON UNIT DISK

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Let $\mathcal{B} := \{f(z) = \sum_{n=0}^{\infty} a_n z^n$ with $|f(z)| < 1$ for all $z \in \mathbb{D}\}$, where $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$. The classical Bohr's inequality [*Proc. Lond. Math. Soc.* s2-13 (1914), 1–5] states that if $f \in \mathcal{B}$, then the associated majorant series $M_f(r) := \sum_{n=0}^{\infty} |a_n| r^n \leq 1$ holds for $|z| = r \leq 1/3$ and the constant $1/3$ cannot be improved. Bohr's original theorem and its subsequent generalizations remain active fields of study, driving investigations in a wide range of function spaces. In this paper, first we establish a generalized Bohr inequality for the class $\mathcal{B}$ by allowing a sequence $\{\varphi_n(r)\}_{n=0}^{\infty}$ of non-negative continuous functions on $[0, 1)$ in the place of $\{r^n\}_{n=0}^{\infty}$ of the majorant series $M_f(r)$ introducing a weighted sequence of non-negative continuous functions $\{\Phi_n(r)\}_{n=0}^{\infty}$ on $[0, 1)$.

A complex-valued function $f(z) = u(x, y)+iv(x, y)$ is called harmonic in Ω if both u and v satisfy the Laplace's equation $\bigtriangledown^2 u = 0$ and $\bigtriangledown^2 v = 0$. It is well-known that under the assumption $g(0) = 0$, the harmonic mapping f has the unique canonical representation $f = h+\overline{g}$, [*Bull. Amer. Math. Soc.* **42** (1936), 689–692.] where h and g are analytic functions in Ω, respectively called, analytic and co-analytic parts of f. A locally univalent harmonic mapping $f = h + \overline{g}$ is sense-preserving whenever its Jacobian $J_f(z) := |f_z(z)|^2 - |f_{\bar{z}}(z)| = |h'(z)|^2 - |g'(z)|^2 > 0$ for $z \in \mathbb{D}$. As a generalization, we obtain a refined version of the Bohr inequality for a certain class $\tilde{G}^0_{\mathcal{H}}(\beta)$ of harmonic mappings. All the results are proved to be sharp.

BOHR PHENOMEON FOR CLASS OF ANALYTIC SELF-MAPS ON THE UNIT DISK AND CLASS OF SUBORDINATION

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Bohr's power series theorem was discovered a century ago in the context of the study of Bohr's absolute convergence problem for the Dirichlet series is now an active area of research for different functions spaces. In 1914, Harald Bohr proved that for every holomorphic function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ on the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ such that $\sum_{n=0}^{\infty} |a_n| r^n \leq 1$ for $|z| = r \leq 1/3$. The constant $1/3$ is famously known as the Bohr radius which is best possible. In this article, we are devoted to the sharp improvement of the classical Bohr's inequality for the class B of analytic self-maps defined on the unit disk $\mathbb{D}$. In addition, we obtain sharp Bohr-Rogosinski type inequalities for the class B with some suitable settings of the area measure of the sub-disk of $\mathbb{D}$. Moreover, we obtain refined sharp Bohr-Rogosinski-type inequalities for the subordination class of univalent and convex function.

SHARP BOUND OF THIRD HANKEL DETERMINANT FOR INVERSE COEFFICIENTS OF STARLIKE FUNCTION OF ORDER 1/2

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Let $\mathcal{A}$ denote the family of all normalized analytic function f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \text{ for } z \in \mathbb{D} := \{z :\in \mathbb{C} : |z| < 1\}.$$

Finding sharp bounds for coefficients for various classes of univalent functions becomes a subject of great interest in geometric function theory. The problem of finding sharp bounds for 3-rd Hankel determinant

$$|H_3(1)(f)| = |2a_2a_3a_4 - a_3^2 - a_4^2 + a_3a_5 - a_2^2a_5|$$

of univalent functions $f \in \mathcal{A}$ is a much more difficult task for study. In this paper, we establish invariance between two functionals of Hankel determinants for some class of functions. The sharp bound for the third Hankel determinant for the coefficients of the inverse function of starlike function of order 1/2 is obtained. Consequently, we deduce that the functionals $|H_3(1)(f)|$ and $|H_3(1)(f^{-1})|$ exhibit invariance on the class $\mathcal{S}^*(1/2)$.

MULTIDIMENSIONAL BOHR RADII FOR VECTOR-VALUED HOLOMORPHIC FUNCTIONS

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In 1914, Harald Bohr obtained the following remarkable result while investigating the absolute convergence problem for ordinary Dirichlet series: Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be a holomorphic function on open unit disk with $|\sum_{n=0}^{\infty} a_n z^n| < 1$. Then $\sum_{n=0}^{\infty} |a_n z^n| \leq 1$ when $|z| \leq 1/3$, and moreover the constant 1/3, called the *Bohr radius* is best possible. Later, several authors have studied the Bohr's power series theorem in higher dimension and introduced the concept of multidimensional Bohr radius: The Bohr radius $K(\Omega)$ of a Reinhardt domain $\Omega \subset \mathbb{C}^n$ is the supremum of all $r \geq 0$ such that for each holomorphic function $f(z) = \sum_{\alpha} a_{\alpha} z^{\alpha}$ on Ω we have $\sup_{z \in r\Omega} \sum_{\alpha} |a_{\alpha} z^{\alpha}| \leq \sup_{z \in \Omega} |\sum_{\alpha} a_{\alpha} z^{\alpha}|$. The main aim of this talk is to answer certain open questions related to the exact values of multidimensional Bohr radii by using the concept of arithmetic Bohr radius for vector-valued holomorphic functions defined in complete Reinhardt domains in $\mathbb{C}^n$. More precisely, we discuss the asymptotic estimates of the arithmetic Bohr radius for holomorphic functions in the unit ball of ℓ_q^n $(1 \leq q \leq \infty)$ spaces with values in arbitrary complex Banach spaces.

VISIBILITY IN QUASIHYPERBOLIC GEOMETRY: A NOTION OF STRICT NEGATIVE CURVATURE IN THE RIEMANNIAN SENSE

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Given a metric space, there are several notions of it being negatively, or in general, nonpositively curved. We consider the underlying metric space being bounded domains in R^n equipped with the quasihyperbolic metric. In this talk, we single out a suitable form of a notion of negative curvature. This notion is called visibility, which, in fact, is a consequence of strict negative sectional curvature of a complete Riemannian manifold, therefore a suitable form of visibility in the context of quasihyperbolic metric can be considered as a weak notion of negative curvature in quasihyperbolic geometry. In this talk, we shall present a general criteria for a domain to be a visibility domain and with this criteria we see that visibility domains form a broad class of domains that includes, for instance, uniform domains, John domains, and domains satisfying quasihyperbolic boundary conditions. Further, we shall see the relationship of visibility with the Gromov hyperbolicity, and we prove that in the case of Gromov hyperbolic domains visibility is necessary and sufficient for the continuous extension of identity map from the Gromov boundary onto the euclidean boundary, which gives a complete solution to an interesting problem in the theory of Gromov hyperbolicity in quasihyperbolic geometry. If time permits we shall discuss the visibility of unbounded domains (which require the concept of end compactification) and its relationship with Gromov hyperbolicity. We shall conclude this talk with some problems for further exploration. This talk is based upon a joint work with Vasudevarao Allu.

SUCCESSIVE COEFFICIENTS FOR FUNCTIONS IN THE SPIRALLIKE FAMILY

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In this talk, we deal with the bounds of successive coefficients $||a_{n+1}| - |a_n||$ for the family of univalent functions that are spirallike functions of non-negative order. This result not only improves a recent result but gives an alternate simple proof for the case of univalent spirallike functions.

ON RETRACTS OF BALANCED DOMAINS OF HOLOMORPHY

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The purpose of this talk is towards characterizing *holomorphic* retracts of balanced pseudoconvex domains $B \subset C^N$. Specifically, we show that every (holomorphic) retract passing through its center (i.e., the origin), is the graph of a holomorphic map over a linear subspace of B. As for retracts not passing through the origin, we obtain the following result: if B is a strictly convex ball with respect to some norm on C^N, and ϱ any holomorphic retraction map on B which is submersive at its center, then $Z = \rho(B)$ is the graph of a holomorphic map over a linear subspace of B. To deal with a case when ∂B may fail to have sufficiently many extreme points, we consider products of strictly convex balls, with respect to various norms and obtain a complete description of retracts passing through the origin, generalizing that of the polydisc due to Heath and Suffrdige and giving a simpler proof as well. The conditions of this result can indeed be verified to be applied to solve a special case with a degeneracy of the union problem, namely: to characterize those Kobayashi corank one complex manifolds M which can be expressed as an increasing union of submanifolds which are biholomorphic to a prescribed homogeneous bounded balanced domain in C^N. Along the way, we discuss examples of non-convex but pseudoconvex bounded balanced domains in which most one-dimensional linear subspaces fail to be holomorphic retracts, in good contrast to the convex case. To go beyond balanced domains, we then first obtain a complete characterization of retracts of the Hartogs triangle. Thereafter, we drop the assumption of boundedness as well, to attain similar characterization results for domains which are neither bounded nor topologically trivial; indeed, for domains with a wide variety of fundamental (or for that matter higher homotopy and homology) groups. We conclude by reporting some results on the retracts of C^2.

BOUNDEDNESS OF GENERALIZED HILBERT-TYPE OPERATORS ON CERTAIN ANALYTIC FUNCTION SPACES

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Abstract

Let $f(z) = \sum_{n=0}^{\infty} a_n z^n$ be any analytic function on the unit disk D. We consider for $\beta \geq 0$, a generalized Hilbert-type operator $\mathcal{H}_\beta$ defined by

$$\mathcal{H}_\beta f(z) = \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \frac{\Gamma(n+\beta+1)\Gamma(n+k+1)}{\Gamma(n+1)\Gamma(n+k+\beta+2)} a_k \right) z^n,$$

in particular, $\beta = 0$ gives the classical Hilbert operator $\mathcal{H}$. We find conditions on the parameter β and establish criteria for the boundedness of $\mathcal{H}_\beta$ within various function spaces such as Hardy spaces H^1, weighted Hardy spaces H^P_α, Bergman spaces with logarithmic weights. We extend some results of Lanuch et.al [1].

Keywords: Generalized Hilbert operators, Hardy spaces, Bergman spaces.

References

[1] B. Lanucha, M. Nowak and M. Pavlović, *Hilbert Matrix operator on Spaces of analytic functions:*, Annales Academiae Scientiarum Fennicae Mathematica, 37, 2012, 161-174.

[2] P. L. Duren, *Theory of H^p spaces*, Academic Press, New York, 1970.

[3] Diamantopoulos E, *Hilbert matrix on Bergman spaces*, Illinois J. Math. 48(3), 2004, 1067-1078.

ON $\mathcal{K}$-MEROMORPHIC FUNCTIONS WITH SIEGEL DISK

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Siegel disks are one of the most interesting objects in the field of complex dynamics. So far most of the study on Siegel disks has been done for polynomials and entire functions. In this work, we use Brujno's theorem to obtain some results that are helpful to find examples of functions in class $\mathcal{K}$ with Siegel disks. We construct several classes of $\mathcal{K}$-meromorphic maps whose elements contain invariant Siegel disks. A result has been given to help us find functions with 2-periodic Siegel disks. These results should serve as a very useful tool specifically to fulfill the relative absence of examples of Siegel disks for non-entire functions. Finally, images of the dynamical plane of some K-meromorphic functions with Siegel disk has been provided for visualizing the results.

BOHR INEQUALITIES FOR CERTAIN TWO PARAMETER CESÁRO AVERAGING OPERATORS

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We define a two parameter family of integral operators $\mathcal{C}^{b,c}$,

$$\mathcal{C}^{b,c}[f](z) = \int_0^z \frac{f(w) * F(1,1;c;w)}{w(1-w)^{b+1-c}} dw,$$

where $*$ denotes the convolution product of power series, $F(a,b;c;z)$ is the classical hypergeometric function and $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is analytic function on the unit disk $\mathbf{D}$ in the complex plane. The main aim of this talk is to discuss sharp Bohr-type radius for integral operators $\mathcal{C}^{b,c}$ when $b > 0, c > 1$ defined on a set of bounded analytic functions in the unit disk.

Keywords: Gaussian hypergeometric function, Bohr inequality.

References

[1] Askey, R., Andrews, G. E., and Roy, R. (1999). Special functions. *Encyclopedia of Mathematics and its Applications*, 71.

[2] Kumar, S., Sahoo, S. K. (2021). Bohr inequalities for certain integral operators. *Mediterranean Journal of Mathematics*, 18(6), 268

[3] Kayumov, I. R., Khammatova, D. M., Ponnusamy, S. (2020). On the Bohr inequality for the Cesáro operator. *Comptes Rendus. Mathématique*, 358(5), 615-620.

[4] Nath, P. K., Naik, S. (2022). Boundedness of $\mathcal{C}^{b,c}$ operators on Bloch spaces. *Korean Journal of Mathematics*, 30(3), 467-474.

DYNAMICS OF TWO ONE-PARAMETER FAMILIES OF MEROMORPHIC FUNCTIONS INVOLVING SINE AND COSINE HYPERBOLIC FUNCTIONS

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In this article, we study the real dynamics of the one-parameter family of entire function $\{f_\lambda(z) = \lambda(1 + \frac{1}{z}) \sinh z$ for $z \in \mathbb{C}$ and λ is a positive real number$\}$ and other is the one-parameter family of meromorphic function $\{g_\lambda(z) = \lambda(1 + \frac{1}{z}) \cosh z$ for $z \in \mathbb{C}$ and λ is a positive real number$\}$, and also, examines their singular values. It is discovered that f_λ and g_λ possess an infinite number of singular values, yet they remain bounded within the complex plane. Intriguingly, it is observed that the majority of these singular values of f_λ and g_λ do not lie on either the real axis or the imaginary axis.

STUDY ON THE RESIDUAL JULIA SETS OF TRANSCENDENTAL SEMIGROUPS

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Our primary goal behind this work is to study the existence of residual Julia sets for semigroups of transcendental entire functions. Here, we define the residual Julia set for transcendental semigroups and give some examples for which residual Julia set is non-empty. It is shown that the residual Julia set for a transcendental semigroup contains the residual Julia set of its elements. Further the residual Julia set is backward invariant when the Julia set is completely invariant for a transcendental semigroup. Also, we prove that the Julia set of a transcendental semigroup is the closure of union of residual Julia sets of its elements. We give some results related to existence of residual Julia set and connectivity of the Fatou set and Julia set. Finally, some results are discussed, concerning presence of buried points in the Julia set.

STUDY ON THE DYNAMICS OF ROOT FINDING MAPS

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In this article, we study the dynamics of certain root finding maps such as Householder's map, Halley's map and Chebyshev's map for various types of functions. Dynamics of Householder's map for degree two polynomials is investigated here. We show that Halley's map has attracting basin corresponding to the roots of any cubic polynomial. We, also find the superattracting extraneous fixed point of Chebyshev's map applied to the Bring-Jerrard polynomial $p_\gamma(z) = z^n + \gamma z + 1$ for different values of parameter γ. For these superattracting extraneous fixed point, Chebyshev's map has attracting basin. Finally, we study the dynamics of Chebyshev's map for the families $\{\lambda \sin \mu z\}$ and $\{\lambda \tan \mu z\}$ where $\lambda, \mu \in \mathbb{C} \setminus \{0\}$.

LOCATION OF ZEROS OF ONE PARAMETER FAMILY OF RATIONAL HARMONIC TRINOMIALS

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It is well known that the Fundamental Theorem of Algebra does not hold true for harmonic polynomials $p(z) = h(z) + \overline{g(z)}$, where $h(z)$ and $g(z)$ are analytic polynomials. In the past few decades, extensive research has been done to find the number of zeros of harmonic polynomials. Recently, Lee [1] introduced rational harmonic trinomials

$$p_c(z) = z^n + \frac{c}{\overline{z}^k} - 1$$

where $n > k > 1$, $n, k \in \mathbb{N}$, $c \in \mathbb{R}^+$ and $\gcd(n, k) = 1$. They investigated zeros of $p_c(z)$ and showed that as c increases, the number of zeros of $p_c(z)$ decreases from $n + k$ to $n - k$. Motive of the present study is to locate those zeros of $p_c(z)$. In particular, for definite ranges of c, we find annular regions each of which contains exactly one zero of $p_c(z)$. Also, we provide pictorial demonstration of the results for some $p_c(z)$.

References

[1] A. Lee. Zeros of a family of complex-valued harmonic rational functions. *Master's Thesis*, Brigham Young University, (2022).

GENERALIZATION OF THE ENESTRÖM-KAKEYA THEOREM THROUGH POLYNOMIAL COEFFICIENTS RELAXATION

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This paper presents a comprehensive exploration of polynomial zero bounds, with a specific focus on the dependence of these bounds on various coefficient constraints and how they evolve as these coefficients change. The study builds upon the prominent Eneström-Kakeya theorem, which states that, *for a polynomial* $P(z) = \sum_{j=0}^{n} a_j z^j$ *of degree* n *satisfying* $a_n \geq a_{n-1} \geq \ldots \geq a_0 > 0$, *all the zeros of* $P(z)$ *fall within* $|z| \leq 1$.

Numerous extensions and generalizations of the Eneström-Kakeya theorem have been documented in the literature. However, it is demonstrated that existing theorems do not consistently yield unique regions for the zeros of polynomials. This emphasizes the need for further investigation into the conditions under which these theorems provide distinct regions for polynomial zeros.

This study offers a detailed analysis of polynomial zero bounds and regions, providing insights into their dependence on coefficient constraints and their evolution. It also highlights the historical context and recent developments in the field, employing mathematical expressions and theorems to support its findings.

THE JULIA SETS OF CHEBYSHEV'S METHOD WITH SMALL DEGREES

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A root-finding method F is a function that associates a given polynomial p with a rational function F_p such that every root z of p is an attracting fixed point of F_p (i.e., $F_p(z) = z$ and $|F_p'(z)| < 1$). The study of rational functions arising as root-finding methods applied to polynomials is an important branch of Complex dynamics which deals with the iterations of rational functions in general. There are several root-finding methods, among them the Newton's method, which is a classical root-finding method, is extensively studied. We consider the Chebyshev's method, which is a third-order convergent root-finding method, is defined as

$$C_p(z) = z - \left[1 + \frac{1}{2}L_p(z)\right]\frac{p(z)}{p'(z)}$$

where p is a non-monomial polynomial of degree at least two. We initiate a dynamical study of C_p for cubic p. If p is cubic then the degree of C_p is found to be $4, 6$ or 7. If p is unicritical (i.e., exactly one finite critical point) or non-generic (i.e., at least one root is multiple) then, it is proved that the Julia set of C_p is connected. The family of all rational maps arising as the Chebyshev's method applied to a cubic polynomial which is non-unicritical and generic (i.e., all roots are simple) is parametrized by the multiplier of one of its extraneous fixed points. Denoting a member of this family with an extraneous fixed point with multiplier λ by C_λ, we have shown that the Julia set of C_λ is connected whenever $\lambda \in [-1, 1]$.

MEROMORPHIC HARMONIC SPIRALLIKE FUNCTIONS WITH A POLE AT THE ORIGIN

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In this talk, we consider the hereditarily meromorphic harmonic λ-spirallike functions, a subclass of the class $\mathcal{MH}$ containing meromorphic harmonic functions with a pole at the origin, denoted as $\mathcal{SP}_{\mathcal{MH}}(\lambda)$. We first discuss sufficient conditions for univalence of functions in class $\mathcal{MH}$. Next, we present several necessary and sufficient conditions for $f \in \mathcal{MH}$ to be in $\mathcal{SP}_{\mathcal{MH}}(\lambda)$. Finally, we give the characterization of hereditarily meromorphic harmonic Archimedean and Hyperbolic spirallike functions.

RADII OF STARLIKENESS OF RATIOS OF ANALYTIC FUNCTIONS WITH FIXED SECOND COEFFICIENTS

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We introduce three classes of analytic functions with fixed second coefficient which are defined using the class $\mathcal{P}(\alpha)$. $\mathcal{P}(\alpha)$ denotes the class of analytic functions of the form $p(z) = 1 + b_1 z + b_2 z^2 + \cdots$ satisfying the condition $Re\{p(z)\} > \alpha$ for some α $(0 \leq \alpha < 1)$ and for all $z \in \Delta = \{z \in : |z| < 1\}$. $\mathcal{P} = \mathcal{P}(0)$ is a well-known class of Carathéodory functions having positive real part. For any two subclasses M and N of family $\mathcal{A}$ of all analytic functions of the form $f(z) = z + a_2 z^2 + \cdots (z \in \Delta)$, the N-radius for the class M, denoted by $R_N(M)$, is the largest number $\rho \in (0, 1)$ such that $r^{-1} f(rz) \in N$, for all $f \in M$ and for $0 < r < \rho$.
The objective is to determine radii such that the three classes are contained in class of starlike functions associated with various regions namely, lemniscate of bernoulli, parabola, exponential, cardiod, sine function, lune, rational function, nephroid and sigmoid. The radii estimated in the present investigation are sharp.