

A SPECTRAL DICHOTOMY FOR COMMUTING M -ISOMETRIES WITH NEGATIVE CORE OPERATOR

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We show that the Taylor spectrum of any pair of commuting m -isometries with negative core operator is either contained in the boundary of the unit bidisc or equal to the closed unit bidisc. Our method of proof relies on a strictly 2-variable fact about the topological boundary of the Taylor spectrum. We also present several applications of the spectral dichotomy to the multivariate Fredholm theory.

New fixed point theorem on complete b-metric space by using Rus contraction mapping

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Abstract

This paper investigates a fixed point over a complete b-metric space for a family of contractive mappings. In this paper, we have discovered new results in the direction of the complete b-metric space by using Rus contraction. Furthermore we establish a common fixed point theorem between two mappings over complete b-metric space. We also provide some non-trivial examples to display the authenticity of our established results.

Keywords: Fixed point; b-metric space; Contraction mapping.

AMS Subject Classification 2020: 46T99; 47H10; 54H25

References

- 1) Bakhtin, I. A., The contraction mapping principle in almost metric spaces, *Funct. Anal.*, 30, Unianowsk, Gos. Ped. Inst., (1989), 26-37.
- 2) Banach, S., Sur les opérations dans les ensembles abstraits et leur application aux équations intégrales, *Fund. Math.* 3(1922), 133-181.
- 3) Bota, M., Molnar, A. and Varga, C., On ekeland's variational principle in b-metric spaces, *Fixed Point Theory* 12 (2011), no. 2, 21-28.
- 4) L. A Rus, Metric space with fixed point property with respect to contractions, *Studia Univ. Babeş-Bolyai* (2006), 115-121.

A BISHOP-PHELPS-BOLLOBÁS TYPE PROPERTY FOR BANACH SPACES WITH RESPECT TO MINIMUM NORM OF BOUNDED LINEAR OPERATORS

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In this paper, we introduce the notion of approximate minimizing property (AMp) for a pair of Banach spaces. This property is a version, with respect to minimum norm of bounded linear operators, of a local Bishop-Phelps-Bollobás type property so called $\mathbf{L}_{o,o}[1]$. Given Banach spaces X, Y and $m(T) = \inf\{\|Tx\| : x \in S_X\}$ the minimum norm for a bounded linear operator $T : X \rightarrow Y$, the pair (X, Y) satisfies AMp for a class $\mathcal{A} \subset \mathcal{B}(X, Y)$ of bounded linear operators if for every $\varepsilon > 0$ and for every $T \in \mathcal{A}$, there exists $\delta(\varepsilon, T) > 0$ such that if $x_0 \in S_X$ satisfies $\|Tx_0\| < m(T) + \delta(\varepsilon, T)$, then there exists $x_1 \in S_X$ satisfying $\|T(x_1) - Tx_0\| < \varepsilon$ such that $\|x_1 - x_0\| < \delta(\varepsilon, T)$. We prove that given any Banach space X (resp. Y), the pair (X, Y) satisfies AMp if and only if Y (resp. X) is finite dimensional. We also investigate some sufficient conditions for a pair (X, Y) of Banach spaces to satisfy AMp for some subclasses of $\mathcal{B}(X, Y)$. One of the above-mentioned results says that if X is reflexive and Y is arbitrary, then (X, Y) satisfies the AMp for the class of all operators from X to Y which are not one-to-one and having closed range. Finally, we define and study uniform ε -approximation of a bounded linear operator in terms of its minimum norm.

Mathematics Subject Classification: Primary 46B20, Secondary 46B04, 46B28.

Keywords: Banach spaces, Bishop-Phelps-Bollobás property, approximate minimizing property, Hausdorff convergence, minimum norm, uniform ε -approximation.

References

- [1] S. Dantas, S.K. Kim, H.J. Lee and M. Mazzitelli, *Local Bishop-Phelps-Bollobás properties* J. Math. Anal. Appl. **468** (2018) 304-323.

ARENS REGULARITY OF THE ORLICZ FIGÀ-TALAMANCA HERZ ALGEBRA

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Let G be a locally compact group. The p -version ($1 < p < \infty$) of the Fourier algebra is called as Figà-Talamanca Herz algebra and is denoted by $A_p(G)$. For $p = 2$, $A_p(G)$ coincides with the Fourier algebra $A(G)$. It is well known that Orlicz spaces are the natural generalization of the classical L^p -spaces. Let $A_\Phi(G)$ be the Orlicz-version of the Figà-Talamanca Herz algebra of G associated with a Young function Φ . As Arens regularity is an important tool to study groups with the help of certain Banach algebras related to it; we show that if $A_\Phi(G)$ is Arens regular, then G is discrete. This generalizes the result by Forrest about the Arens regularity of the $A_p(G)$ algebras. We also show that $A_\Phi(G)$ is finite-dimensional if and only if G is finite. Further, for amenable groups, we show that $A_\Phi(G)$ is reflexive if and only if G is finite, under the assumption that the associated Young function Φ satisfies the MA-condition.

TWO TRUNCATED MATRIX MOMENT PROBLEMS

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The classical truncated moment problem, also known as the *Hamburger moment problem* on the real line, asks whether a real finite sequence s_0, s_1, \dots, s_{2n} can be represented as the sequence of successive moments of some positive measure μ , that is, in the form

$$s_k = \int_{-\infty}^{+\infty} x^k d\mu \quad (k = 0, 1, 2, \dots, 2n).$$

In this talk, we shall focus on a matrix version of the above truncated moment problem in two different settings: on the real line and on the circle. We shall show that the de Branges spaces are a natural tool for studying truncated matrix moment problems. Along the way, we shall find links to different types of matrices like Toeplitz and Hankel matrices, as well as matrix polynomials. Ultimately, we aim to provide a comprehensive description of a family of solutions to the above problem.

MULTIVALUED GERAGHTY CONTRACTIONS AND THEIR BEST PROXIMITY POINTS

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Sometimes, we encounter scenarios where we need to employ an approximate optimal solution for equations of the form $g(t) = t$, even when g is not a self-map, due to the absence of an exact solution for that equation. The presence of such optimal solutions is guaranteed by best proximity point theorems. In this work, we introduce the concept of multivalued Geraghty contraction (MVG C) within a complete metric space and establish the corresponding best proximity point (BPP) result. Our finding expands upon Geraghty's renowned result on fixed points.

KOWALSKI-SŁODKOWSKI THEOREM FOR CONDITION SPECTRUM

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Kowalski and Słodkowski gave a characterization of multiplicative linear functionals on complex Banach algebras \mathcal{A} without the linearity assumption. The characterization is: Let $\varphi : \mathcal{A} \rightarrow \mathbb{C}$ satisfying $\varphi(0) = 0$ and $\varphi(x) - \varphi(y) \in \sigma(x - y)$ for every $x, y \in \mathcal{A}$. Then φ is multiplicative and linear. We try to generalize the Kowalski-Słodkowski theorem for approximately multiplicative functionals on commutative Banach algebra using condition spectrum and see how far the generalization can be made. Likewise we also characterize the multiplicative linear functional using exponential spectrum or in general, some special Ransford spectrum.

TOEPLITZ OPERATORS ON THE N -DIMENSIONAL HARTOGS TRIANGLE

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We formally introduce and study Toeplitz operators on the Hardy space of the n -dimensional Hartogs triangle

$$\Delta_0^n := \{(z_1, z_2, \dots, z_n) \in \mathbb{C}^n : |z_1| < |z_2| < \dots < |z_n| < 1\}.$$

We find a precise relation between these operators and the Toeplitz operators on the Hardy space of the unit polydisc \mathbb{D}^n . As an application, we deduce several basic properties of these operators from their polydisc counterparts.

NEARLY INVARIANT BRANGESIAN SUBSPACES

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Abstract:

In this talk, I will discuss my work on nearly invariant subspaces. Nearly invariant subspaces for backward shift operator on Hardy space $H^2(\mathbb{D})$ were first discussed by Hitt in his work on invariant subspaces of H^2 on an annulus. Since then, this work has been extended in many directions. Here, I will present our extensions of Hitt's theorem as well as its generalizations to the de Branges setting. This is the joint work with Dr. Sneh Lata and Dr. Dinesh Singh.

CONTROLLABILITY OF SECOND ORDER DIFFERENTIAL SYSTEMS WITH NONLINEAR CONTROL

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This work presents the study on a family of second order differential systems with nonlinear control in Hilbert spaces. The results are obtained through the linear approximation of the nonlinear system. By using the perturbation theory of linear operators by time-dependent bounded linear operator, the mild solution is obtained for the linearized control system. The mild solution of the main system is obtained by the variation of constants formula. Then, sufficient conditions for controllability of the nonlinear control system have been established under the assumption that the linearized control system is controllable. The results are applicable to real life phenomena described by partial differential equations.

Chukwu [1, 2] established the controllability of nonlinear delay systems. The concept of perturbation of a linear operator by time-dependent operator was presented by Lutz [3] for second order evolution equations. Matsumoto [4] established the time-dependent perturbation theory for the first order evolution equation. These are motivation to obtain the mild solution of the linearized control system which is nonautonomous.

References

- [1] E. N. Chukwu and S. M. Lenhart. Controllability questions for nonlinear systems in abstract spaces. *Journal of Optimization Theory and Applications*, 68(3):437–462, 1991.
- [2] E. N. Chukwu. Nonlinear delay systems controllability. *Journal of Mathematical Analysis and Applications*, 162(2):564–576, 1991.
- [3] Dieter Lutz. On bounded time-dependent perturbations of operator cosine functions. *Aequationes Mathematicae*, 23:197–203, 1981.
- [4] T. Matsumoto. Time-dependent nonlinear perturbations of integrated semigroups. *Nihonkai Mathematics Journal*, 7:1–28, 1996.

ROKHLIN DIMENSION: PERMANENCE PROPERTIES AND IDEAL SEPARATION

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We are trying to understand the structure of the crossed product C^* -algebra using the notion of Rokhlin dimension for group actions. For compact groups Gardella, Hirshberg and Phillips have given a number of permanence properties. In this talk we are going to see the analogous notion for countable discrete residually finite groups. First we give a definition which is equivalent to the original definition given by Szabó, Wu and Zacharias. We then prove permanence properties. Finally, we discuss the ideal structure of the crossed product.

OPERATOR SPACE STRUCTURES OF NON-COMMUTATIVE UNITARY C^* -ALGEBRA

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The non-commutative unitary C^* -algebra U_n^{nc} , introduced by L. Brown is universal unital C^* -algebra generated by elements u_{ij} , $1 \leq i, j \leq n$ satisfying the relations which make $[u_{ij}]$ a unitary matrix. It is a universal C^* -algebra with universal property that if v_{ij} , $1 \leq i, j \leq n$ are elements in a C^* -algebra A with $[v_{ij}]$ being a unitary matrix in $M_n(A)$, then there exists a unique unital $*$ -homomorphism $\pi : U_n^{nc} \rightarrow A$ such that $\pi(u_{ij}) = v_{ij}$. In this talk, we will discuss about operator space structure of the standard generators of U_n^{nc} . In particular, we will talk about the maximal and minimal operator space structure of the standard generators u_{ij} , $1 \leq i, j \leq n$. We will then talk about the universal C^* -algebraic structure corresponding to the operator space of U_n^{nc} . This is an ongoing joint work with Dr. Issan Patri and Sayan Kansa Banik.

THE WEYL TRANSFORM OF A MEASURE

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(1) Let $\mathcal{H} = L^2(\mathbb{R}^n)$, and let $\mathcal{B}(\mathcal{H})$ be the set of bounded operators on \mathcal{H} . If λ is a finite measure on \mathbb{R}^{2n} , the *Weyl Transform* of λ is the operator $W(\lambda) \in \mathcal{B}(\mathcal{H})$ defined by

$$(W(\lambda)\varphi)(t) = \int e^{\pi i(x \cdot y + 2y \cdot t)} \varphi(t + x) d\lambda(x, y).$$

Suppose S is a compact connected smooth hypersurface in \mathbb{R}^{2n} , $n \geq 2$, whose Gaussian curvature is positive everywhere. By a *smooth measure on S* we mean a measure of the form $\mu = \psi\sigma$ where σ is the measure on S induced by the Lebesgue measure on \mathbb{R}^{2n} and ψ is a smooth function on \mathbb{R}^{2n} whose support intersects S in a compact set. Then $W(\mu)$ is a compact operator, and if $p > n \geq 6$ then $W(\mu)$ belongs to the p -Schatten class.

(2) Let $H_{2n+1} = \{(x, y, z) | x, y \in \mathbb{R}^n, z \in \mathbb{C}, |z| = 1\}$ denote the (reduced) Heisenberg group. According to the *Stone-von Neumann Theorem*, there is a unique irreducible unitary representation ρ of G such that

$$\rho(0, 0, z) = zI.$$

There is an action of \mathbb{R}^{2n} on the p -Schatten class called *quantum translation* which is defined by

$$(x_1, y_1) \cdot A = \rho(x_1, y_1, 1)A\rho(x_1, y_1, 1)^{-1}.$$

As an application of the result (1), we can show that there exist Schatten class operators with linearly dependent quantum translates.

**A GENERALIZED GKZ THEOREM AND
KOWALSKI-SŁODKOWSKI THEOREM FOR
REPRODUCING KERNEL HILBERT SPACE**

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The Gleason-Kahane-Zelazko theorem is one of the important theorems in Banach algebra, which tells us about the multiplicativity of a linear functional. Cheng Chu, Michael Hartz, Javad Mashreghi and Thomas Ransford gave a similar result for reproducing kernel Hilbert space \mathcal{H} with normalized complete Pick kernel. We will discuss about generalizing linearity in the hypothesis of that theorem. We will also discuss about generalizing the Kowalski-Słodkowski theorem for RKHS with complete Pick kernel.

ON SOME GEOMETRIC CONSTANTS IN A BANACH SPACE

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During this talk we describe how the notion of isosceles orthogonality plays a crucial role regarding the attainment of James constant, Schäffer constant and the local version of it. We further study the attainment of James constant in a two-dimensional polyhedral Banach space X . We show that there exists an extreme point x on the unit ball of X such that James constant attains its value on x . We also establish the relation between the approximate isosceles orthogonality and the notion of modulus of convexity in a Banach space.

This talk is based on the article [1].

References

- [1] Sain, D., Ghosh, S., Paul, K., *On isosceles orthogonality and some geometric constants in a normed space*, Aequat. Math., **97** (2023), 147-160.

INTERPOLATION RESULTS FOR CONVERGENCE OF IMPLICIT EULER SCHEMES WITH ACCRETIVE OPERATORS

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In this paper, we study the nonlinear abstract Cauchy problem with an m -accretive operator. If the initial value and the right-hand side lie in some interpolation sets with parameter α , then we obtain the rate of convergence $O(|\pi|^{\alpha/2})$ for solutions of the corresponding implicit Euler schemes and for the Euler solution as the limit, we obtain Hölder continuity of exponent α . This regularity result can be generalized to other notions of interpolation. This is a joint work with Johann Beurich from Technical university of Dresden.

References

- [1] V. Barbu. *Nonlinear differential equations of monotone types in Banach spaces*. Springer Monographs in Mathematics. Springer, New York, 2010.
- [2] Ph. Bénéilan, M. G. Crandall, and A. Pazy. *Nonlinear Evolution Equations Governed by Accretive Operators*. Book manuscript, 1999.
- [3] J. Beurich. *Euler schemes for accretive operators on Banach spaces*. PhD thesis, TU Dresden, Dresden, 2023
- [4] D. Brézis. *Interpolation et opérateurs non linéaires*. Thèse de doctorat, Université Paris VI, 1974.
- [5] R. Chill, P. Sharma, and S. Srivastava. *Real interpolation of functions with applications to accretive operators on Banach spaces*. arXiv:2206.13400 (2022).
- [6] M. G. Crandall and L. C. Evans. On the relation of the operator $\partial/\partial s + \partial/\partial \tau$ to evolution governed by accretive operators. *Israel J. Math.* 21, 261-278 (1975).

VECTOR-VALUED PROPERTIES OF WEYL TRANSFORM

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The notion of Fourier type with respect to a locally compact abelian group was introduced by J. Peetre for \mathbb{R} and by Milman for general locally compact abelian groups. We introduce and study the concept of Weyl type/cotype with respect to a locally compact abelian group in the setup of operator spaces. We investigate properties like the Hausdorff-Young inequality and Plancherel theorem in vector-valued settings and show that analogue of these properties still holds for the Weyl transform of vector-valued functions. Contrary to this, the Riemann-Lebesgue lemma fails for the Weyl transform of vector measures. Eventually, we also provide a sufficient condition for a vector measure to satisfy the Riemann-Lebesgue lemma.

Extreme contractions on some special Banach spaces

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We further study finite-dimensional subspaces of ℓ_∞ which are of the form ℓ_∞^n for some $n \geq 2$. As an application of the results we compute the number of extreme contractions for a class of the space of bounded linear operators. In particular we find the number of extreme contractions of $\mathbb{L}(\mathbb{X}, \ell_\infty^n)$, where \mathbb{X} is a finite-dimensional polyhedral space. This talk is based on the articles [1, 2].

REFERENCES

- [1] S. Sohel, D. Sain and K. Paul, *On subspaces of ℓ_∞ and extreme contractions in $L(X, \ell_\infty)$* , Monatsh. Math., (2023). <https://doi.org/10.1007/s00605-023-01867-6>.
- [2] D. Sain, S. Sohel and K. Paul, *Extreme contractions on finite-dimensional Banach spaces*, Colloq. Math., **172**(2023) 65-83.