HANDLE DECOMPOSITION FOR A LARGE CLASS OF COMPACT ORIENTABLE PL 4-MANIFOLDS

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It is known that any PL 4-manifold admits a handle decomposition. In this paper, we study compact orientable PL 4-manifolds having empty or connected boundary with fundamental group of rank 1. We give a particular type of handle decomposition for such manifolds using the graph theoretic notion for manifolds, **Crystallization**. The class concerned here consists of manifolds admitting weak semi-simple crystallization. We construct a handle decomposition such that the number of 2-handles depends on the second betti number of the manifold.

ON A CONSTRUCTION OF SOME HOMOLOGY D-MANIFOLDS

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The g-vector of a simplicial complex contains a lot of information about the combinatorial and topological structure of that complex. Several classification results regarding the structure of normal pseudomanifolds and homology manifolds have been established concerning the value of g_2. It is known that when $g_2 = 0$, all normal pseudomanifolds of dimensions at least three are stacked spheres. In the cases of $g_2 = 1$ and 2, all homology manifolds are polytopal spheres and can be obtained through retriangulation or join operations from the previous ones. In this article, we provide a combinatorial characterization of the homology d-manifolds, where $d \geq 3$ and $g_2 = 3$. These are spheres and can be obtained through operations such as joins, some retriangulations, and connected sums from spheres with $g_2 \leq 2$. Furthermore, we have presented a structural result on prime normal d-pseudomanifolds with $g_2 = 3$. Our results, in conjunction with results by Swartz, classify (combinatorially) all the normal 3-pseudomanifolds with $g_2 = 3$.

Rapid fluctuations of a curve under chaotic maps

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Abstract

The behaviour under the iterations of a Devaney chaotic map f on a space X changes drastically. That means, at least for the maps on a metric space X, the orbits of nearby points can be far away after some applications of f. This phenomena is known as the sensitive dependence on initial conditions. In [1], the authors proved that if an interval map f has sensitive dependence on initial conditions then the total variation of f^n on any subinterval tends to infinity as $n \to \infty$.

The notion of topological entropy measures the exponential growth rate of the number of distinguishable orbits, which is invariant under topological conjugacy. There is a relation between topological entropy and Devaney choas at least for the case of graph maps ([3]). In 1980, Misiurewicz and Szlenk connected the notion of topological entropy and the growth of total variation for an interval map ([2]).

In this research problem we generalize the result of [2]. In particular, we prove that any non constant curve on the real line has rapid fluctuations under the iterations of a transitive map on the real line and the same result is also proved for a Devaney chaotic map on a graph.

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EXPLORING EXPANSIVITY AND ITS IMPLICATION

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In the theory of dynamical systems expansivity and shadowing property play a crucial role in determining behaviour of a dynamical system. In this talk, we will discuss notions of expansivity, shadowing property and their consequences including decomposition theorem. We focus on bi-asymptotically c-expansive maps on metric spaces and its relationship with other variants of expansivity. Further, we present some of our recent results relating bi-asymptotic c-expansivity and shadowing property on compact metric spaces and discuss some recent problems.

MAPS BETWEEN RELATIVELY HYPERBOLIC BOUNDARIES

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In 1987, M. Gromov introduced the notion of hyperbolic metric spaces. A geodesic metric space is said to be hyperbolic if there exists a $\delta \geq 0$ such that for any geodesic triangle $\triangle$, each side of $\triangle$ is contained in a closed δ-neighborhood of the union of the other two sides. The Gromov boundary ∂X of a proper geodesic hyperbolic metric space X is the set of equivalent classes of geodesic rays starting from a fixed basepoint $x_0 \in X$, where two such rays are equivalent if their Hausdorff distance is finite. A quasi-isometry is a coarsely bi-Lipschitz map between two metric spaces. Quasi-isometry between two hyperbolic metric spaces induces a homeomorphism between their Gromov boundaries. However, the converse is not true. There exist two hyperbolic groups, due to Bourdon, with homeomorphic Gromov boundaries, but they are not quasi-isometric.

In 1996, F. Paulin proved that if Gromov boundaries of two hyperbolic groups are homeomorphic and quasi-Möbius equivalent, then they are quasi-isometric to each other. A quasi-Möbius map is defined in terms of cross ratios. Corresponding to a triangle $\triangle(a, b, c)$ in $X \cup \partial X$, there exists a point $p_{abc} \in X$ (called a quasiprojection) such that distance of p_{abc} to each side of the $\triangle(a, b, c)$ is bounded in terms of the hyperbolicity constant of X. Given any four distinct points $a, b, c, d \in \partial X$, a cross-ratio $[a, b, c, d]$ roughly estimates the distance between barycenters of the triangles $\triangle(a, b, c)$ and $\triangle(a, c, d)$. A quasi-Möbius map $f : \partial X \to \partial Y$ roughly compares the cross ratios $[a, b, c, d]$ and $[f(a), f(b), f(c), f(d)]$.

Relatively hyperbolic groups were first introduced by M. Gromov, and it is a generalisation of hyperbolic groups. Let G be a finitely generated group and H be a subgroup of G. We say G is hyperbolic relative to H if the space G^h obtained by attaching 'combinatorial horoballs' to each of the left cosets of H in a Cayley graph of G is a hyperbolic metric space. The boundary ∂G^h is called the relative hyperbolic boundary of G. Our goal is to extend Paulin's results to relatively hyperbolic groups. Given two relatively hyperbolic groups, we have proved that if their relative hyperbolic boundaries satisfy some conditions similar to quasi-Möbius equivalence, then the groups are quasi-isometric to each other. In this talk, after introducing basic notions, we will give a brief sketch of our result.

DYNAMICS OF ACTIONS OF AUTOMORPHISMS OF DISCRETE GROUPS ON CERTAIN COMPACT SPACES

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For a locally compact Hausdorff group G and the compact space Sub_G of closed subgroups of G endowed with the Chabauty topology, we study the dynamics of actions of automorphisms of G on Sub_G in terms of distality and expansivity. We prove that an infinite discrete group G, which is either polycyclic or a lattice in a connected Lie group, does not admit any automorphism which acts expansively on Sub^c_G, the space of cyclic subgroups of G, while only the finite order automorphisms of G act distally on Sub^c_G. For an automorphism T of a connected Lie group G which keeps a lattice Γ invariant, we compare the behaviour of the actions of T on Sub_G and Sub_Γ in terms of distality. Under certain necessary conditions on the Lie group G, we show that T acts distally on Sub_G if and only if it acts distally on Sub_Γ.

ON THE CHARACTERIZATION OF COSYMPLECTIC MANIFOLDS AND RIGIDITY OF THE SASAKIAN STRUCTURE

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We introduce new metric structures on a smooth manifold (called "weak" structures) that generalize the almost contact, Sasakian, cosymplectic, etc. metric structures (φ, ξ, η, g) and allow us to take a fresh look at the classical theory and find new applications. This assertion is illustrated by generalizing several well-known results. It is proved that any Sasakian structure is rigid, i.e., our weak Sasakian structure is homothetically equivalent to a Sasakian structure. It is shown that a weak almost contact structure with parallel tensor φ is a weak cosymplectic structure and an example of such a structure on the product of manifolds is given. Conditions are found under which a vector field is a weak contact vector field.

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SOME IMPLICATIONS OF RELATIVE COVERING AXIOMS

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In 1989 Arhangel'skii and Gannedi in their fundamental study of relative topological spaces defined relative paracompactness and relative Lindelöfness. Various versions of these notion have been studied later on like relative 1-paracompact, relative nearly paracompact spaces, relative Aull paracompact. In 2002, Hoshina and Yamazaki defined another relative covering axioms called relative collectionwise normality. Later on in 2005 Grabner et al. studied relative star normal type space. Various relative separation axioms via different types of open sets have been defined and studied in the past. In this paper we established and studied the relationships between some relative separation axioms and relative covering axioms.

THE BRYLINSKI BETA FUNCTION OF A DOUBLE LAYER

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An analogue of Brylinski's knot beta function is defined for a compactly supported (Schwartz) distribution T on d-dimensional Euclidean space. This is a holomorphic function on a right half-plane. If T is a (uniform) double-layer on a compact smooth hypersurface, then the beta function has an analytic continuation to the complex plane as a meromorphic function, and the residues are integrals of invariants of the second fundamental form. The first few residues are computed when $d = 2$ and $d = 3$.

FILLING WITH SEPARATING CURVES

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A pair (α, β) of simple closed curves on a closed and orientable surface S_g of genus g is called a filling pair if the complement is a disjoint union of topological disks. If α is separating, then we call it as separating filling pair. Here, we discuss a necessary and sufficient condition for the existence of a separating filling pair on S_g with exactly two complementary disks. We study the combinatorics of the action of the mapping class group $\mathrm{Mod}(S_g)$ on the set of such filling pairs. Furthermore, we construct a Morse function $\mathcal{F}_g$ on the moduli space $\mathcal{M}_g$ which, for a given hyperbolic surface X, outputs the length of shortest such filling pair with respect to the metric in X. We show that the cardinality of the set of global minima of the function $\mathcal{F}_g$ is the same as the number of $\mathrm{Mod}(S_g)$-orbits of such filling pairs.

TRAVERSABLE WORMHOLES SUPPORTED BY DARK MATTER AND MONOPOLES WITH SEMI-CLASSICAL EFFECTS [1]

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Wormholes describe an incredible spacetime structure of joining two separate parts of the same world or universes. Einstein field equations perform like the pillar towards the concept of wormholes. Einstein and Rosen endorse the concept of spacetime deformation through a geometrical approach as elementary particles like electrons are governed by a geometrical structure now known as Einstein Rosen Bridge (ERB). The ERB was indicated to be unstable for the topological defects mainly by global monopole, which could be a part of galaxies with spiral arms having dark matter. It is true that the study of wormhole geometry created an enthusiastic stir among theoretical researchers over the last few years. As a result, the wormholes are found in the galactic halo region supported by the NFW, URF, and Isothermal DM density profiles. It is argued that topological defects are the base theme to discuss the role of dark matter in cosmology by invoking the Einasto profile as a plausible explanation for density perturbations that seeded galaxy formation and proposed one of the interesting topological defects by global monopole with semiclassical effects in the framework of a 1 + 3 dimension.

In this article, for the first time, we have introduced a new traversable wormhole explication of Einstein's field equations supported by the profile of Einasto Dark Matter densities and global monopole charges along with semi-classical effects in the local universe as the galactic halo. The Einasto DM density profile produces a suitable shape function that meets all the requirements for presenting the wormhole geometries. The Null Energy Condition (NEC) is violated by the obtained solution with different redshift functions i.e. the Einasto profile representing DM candidate within the wormholes gives the fuel to sustain these wormhole structures in the galactic halo. Moreover, the reported wormhole geometries are getting asymptotically flat and non-flat depending only on the choices of redshift function whereas all the wormhole structures are maintaining their balance of equilibrium under the action of different forces.

[1] This paper has already been published in The European Physical Journal C (https://doi.org/10.1140/epjc/s10052-023-11456-4).

NON STAR ROTHBERGER PROPERTIES EXPERIENCING THE RESIDUALS IN AN IDEAL

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This article presents the concept of the star-Rothberger property modulo an ideal by combining the star operator with the notion of an ideal Rothberger space. Subsequently, an analysis of some topological aspects of this property is conducted. By establishing connections among several topological characteristics that exhibit patterns akin to the star-Rothberger space, we strengthen the underlying notion. To elucidate the distinctions among various interconnected topological features, we furthermore provide many instances that serve as counterexamples. This study examines many features pertaining to preservation inside subspaces and functions. Finally, we propose a method to represent the ideal star-Rothberger space using families of closed sets, using certain changes to the SSI^{I} feature.

A GENERALIZATION OF A RESULT OF MINAKSHISUNDARAM AND PLEIJEL

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Let M be a compact Riemannian manifold of dimension m, N a compact submanifold of M, and $\psi \in C^\infty(M)$. Suppose

$$-\Delta\phi_j = \lambda_j \phi_j, \qquad j = 1, 2, \ldots$$

is the spectral decomposition of the Laplace-Beltrami operator of M.

Let

$$c_j = \int_N \phi_j \psi \, d\nu,$$

where ν denotes the Riemannian measure on N.

In 1949, Minakshisundaram and Pleijel proved that if N is a point (and $\psi = 1$) then

$$\sum_{\lambda_j < T} c_j^2 \sim \frac{T^{m/2}}{(2\sqrt{\pi})^m \Gamma(\frac{m}{2} + 1)}, \qquad T \to \infty.$$

We generalize this result.

ON DYNAMICAL NOTIONS OF NON-AUTONOMOUS SYSTEMS

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Dynamical systems theory has long been used to investigate various physical or natural phenomena around us. Many of these processes have been studied and simulated using discrete or continuous systems. Usually, a discrete autonomous dynamical system is denoted by a pair (X, f) where X is a compact metric space and f is a continuous self-map on X. As a result, the governing rule f for a dynamical system is assumed to remain constant throughout time. While these investigations have provided good predictions of the underlying systems, even more, precise approximations can be achieved by allowing the governing rule to vary with time. Such systems are known as non-autonomous discrete dynamical systems. A non-autonomous dynamical system generated by the family $\mathbb{F} = \{f_n : n \in \mathbb{N}\}$ is denoted by the pair $(X, \mathbb{F})$. In such a setting orbit of any point x_0 can be visualized as the iterative image under the ordered set $\{\ldots, f_2^{-1}, f_1^{-1}, I, f_1, f_2, \ldots, \}$. In short, *orbit* of any point $x_0 \in X$ is defined as the set $O(x) = \{\omega_n(x) : n \in \mathbb{Z}\}$ where $\omega_n = \left\{ \begin{array}{ll} f_n \circ f_{n-1} ... \circ f_1 & : n \geq 1, \\ f_n^{-1} \circ f_{n-1}^{-1} ... \circ f_1^{-1} & : n < 0. \end{array} \right\}$ and $\omega_0 = Id_X$.

In this talk, we will discuss topological dynamics of non-autonomous dynamical systems generated by a commutative family of homeomorphisms. In particular, we will discuss various dynamical properties of non-autonomous dynamical systems such as equicontinuity, minimality, almost periodicity and present some recent results. In our recent work we observed that a non-autonomous system need not have an almost periodic point contrary to an autonomous dynamical system and the same is also true for a minimal or equicontinuous system. We will introduce notion of orbital hulls in non-autonomous dynamical systems and relate it with the dynamics of system $(X, \mathbb{F})$. We will give a sufficient and necessary condition for the system to be minimal in terms of orbital hull. We will also relate almost periodicity of the orbital hull of a point to equicontinuity of the non-autonomous dynamical system. We will show that any minimal system generated by commutative family is either equicontinuous or has a dense set of sensitive points.