Primitive idempotents in rational group algebras of monomial groups

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The computation of the primitive idempotents of a group ring gives control of the representations in the base field inside the group ring. Moreover, a complete set of orthogonal primitive idempotents gives us enough information to compute all one-sided ideals of the group ring up to conjugation. We present a method to explicitly compute a complete set of orthogonal primitive idempotents in a simple component with Schur index 1 of a rational group algebra for a class of monomial groups. We will exemplify the theoretical constructions with a detailed concrete example to illustrate the theory.

RATIONAL REPRESENTATIONS AND RATIONAL GROUP ALGEBRA OF SOME CLASSES OF P-GROUPS

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Let G be a finite group, and let $\chi \in \mathrm{Irr}(G)$. Then $\Omega(\chi)= m_{\mathbb{Q}}(\chi)\sum_{\sigma\in \mathrm{Gal}(\mathbb{Q}(\chi)/\mathbb{Q})}\chi^{\sigma}$ corresponds to an irreducible $\mathbb{Q}$-representation ρ of G, where $m_{\mathbb{Q}}(\chi)$ represents the Schur index of χ over $\mathbb{Q}$. Conversely, if there exists an irreducible $\mathbb{Q}$-representation ρ of G, then there is a character $\chi \in \mathrm{Irr}(G)$ such that $\Omega(\chi)$ is the character of ρ. In this talk, we will discuss the algorithm to compute an irreducible rational matrix representation of a p-group G that affords the chracter $\Omega(\chi)$. Furthermore, we will discuss some combinatorial formulas to determine the Wedderburn decomposition of rational group algebras for some classes of p-groups.

COMPATIBILITY OF KAZHDAN AND BRAUER HOMOMORPHISM

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The local Langlands correspondence relates the set of irreducible smooth representations of the group of rational points of a reductive group over a local field–with the representation theory of its Weil–Deligne group. For a split reductive group $\mathbf{G}$ defined over $\mathbb{Z}$, Kazhdan conjectured a link between the local Langlands correspondences for $\mathbf{G}(F)$ and $\mathbf{G}(F')$, where F is a p-adic field and F' is a non-Archimedean equal characteristic local field which are sufficiently close. Kazhdan's approach is via isomorphisms between the respective Hecke algebras, with integral coefficients, $\mathcal{H}(\mathbf{G}(F), K)$ and $\mathcal{H}(\mathbf{G}(F'), K')$, for some specific choice of open-compact subgroups K and K' of $\mathbf{G}(F)$ and $\mathbf{G}(F')$ respectively. Say L is a non-Archimedean local field with residue characteristic p and assume that K is a nice compact open subgroup of $\mathbf{G}(L)$–for instance a congruence subgroup of positive level. When $\mathbf{G}(L)$ has an automorphism, denoted by σ, of prime order $l \neq p$ such that $\sigma(K) = K$, Treumann–Venkatesh define an $\overline{\mathbb{F}_l}$-algebra homomorphism between $\mathcal{H}(\mathbf{G}(L), K) \otimes \overline{\mathbb{F}_l}$ and $\mathcal{H}(\mathbf{G}^\sigma(L), K^\sigma) \otimes \overline{\mathbb{F}_l}$, which is called the Brauer homomorphism. The Brauer homomorphism is conjectured to be compatible with Langlands functoriality principle. In this talk, we discuss about compatibility of Brauer homomorhism and Kazhdan homomorphism in the Base change setting. Under some finiteness condition on the Tate cohomology $\widehat{H}^i(\pi)$ (which we prove for the group GL_n), we try to give an application of the above compatibility result to the linkage principle.

ON KRONECKER COEFFICIENTS FOR REPRESENTATIONS OF FINITE GROUPS

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Understanding Kronecker coefficients, which are the multiplicities of irreducible representations in the tensor product of two irreducible representations for a finite group, poses a significant challenge. A group, denoted as G, is said to have a multiplicity-free tensor product when all of its Kronecker coefficients are at most one. Additionally, a finite group is called doubly real if, for every pair of elements, there exists a element that can conjugate the pair to their respective inverses.

In this presentation, we prove that doubly real groups possess a multiplicity-free tensor product. We explore this notion of doubly reality on groups and prove that no finite simple group other than C_2 is doubly real.

We make an interesting observation that the sum of the squares of Kronecker coefficients of G is equal to the number of simultaneous conjugacy classes of pairs in the group. Generally, there is no such straightforward interpretation for the sum of all Kronecker coefficients of G, with the exception of Symmetric groups, as discussed in works by Joseph Ben Geloun and Sanjaye Ramgoolam.

For any finite group G, we demonstrate that a weighted sum of Kronecker coefficients corresponds to the number of doubly real simultaneous conjugacy classes of pairs in G. The weight assigned to a triplet of irreducible representations is determined by the product of their respective Frobenius-Schur indices, which can be either 1, 0, or -1.

Our result extend Geloun-Ramgoolam's work to total orthogonal groups, where all Frobenius-Schur indices are equal to 1. An important implication of our theorem is that if G is a real simple non-abelian group, then it cannot have multiplicity-free tensor product.