A Q- SERIES IDENTITY OF UCHIMURA AND ITS NUMEROUS GENERALIZATIONS

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In 1981, Uchimura rediscovered an interesting q-series identity of Ramanujan, whose one side is the generating function for the divisor function $d(n)$. Mainly, he proved the following identity. For $|q| < 1$,

$$\sum_{n=1}^{\infty} nq^n(q^{n+1})_\infty = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}q^{\frac{n(n+1)}{2}}}{(1-q^n)(q)_n} = \sum_{n=1}^{\infty} \frac{q^n}{1-q^n}.$$

Over the years, this identity has been generalized by many mathematicians in different directions. Uchimura himself in 1987, Dilcher (1995), Andrews-Crippa-Simon (1997), and recently Gupta-Kumar (2021) found a generalization. Any generalization of the right most expression of the above identity, we call as "**divisor-type sum**", whereas a generalization of the middle expression we say "**Ramanujan-type sum**", and generalization of the left most expression as "**Uchimura-type sum**". In this talk, we will discuss these generalizations and present a unified theory.

FOURTH POWER MEAN OF THE GENERAL S-DIMENSIONAL KLOOSTERMAN SUMS MOD P

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In this talk, we discuss about an asymptotic formula for the fourth power mean of the general s-dimensional Kloosterman sum mod p. Initial works towards this direction is due to Zhang Wenpeng. He used analytic methods and estimate of character sums to prove the fourth power mean values of general 2 and 3 dimensional Kloosterman sums (2017-2019). Later with R. Barman we improved the error term for the 3 dimensional case of Zhang's result and eventually got an asymptotic formula for the 4-dimensional case. We used a result of P. Deligne, which counts the number of $\mathbb{F}_p$-points on the surface

$$(x-1)(y-1)(z-1)(1-xyz) - uxyz = 0, \; u \neq 0,$$

and then took an average of the error terms over u to prove the asymptotic formula. We also found the number of solutions of certain congruence equations mod p which are used to prove our main result. Very recently, with A. Haldar we proved an asymptotic formula for the fourth power mean of general s-dimensional hyper-Kloosterman sum. We found the number of solutions of certain congruence equations mod p which play an integral part to prove our main result. We used estimates for character sums and analytic methods to prove our theorem.

EXPONENTIAL SUMS WITH MULTIVARIABLE RATIONAL FUNCTIONS AND THEIR APPLICATIONS TO EQUIDISTRIBUTION OF SOLUTIONS OF QUADRATIC CONGRUENCES

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In the first part of the talk, we will develop a method of evaluating general exponential sums with rational amplitude functions for multiple variables which complements the works of Todd Cochrane. Then we will see an application of it to get an equidistribution result of solutions of quadratic congruences modulo prime power. Let p a fixed odd prime and $Q(x_1,...,x_{n+1}) = \sum_{k=1}^{n} x_k^2 - x_{n+1}^2$ be a fixed quadratic form in $\mathbb{Z}[x_1,...,x_{n+1}]$. Let $(x_{0_1},...x_{0_{n+1}})$ be a fixed point in $\mathbb{Z}^{n+1}$. We study the behavior of solutions $(x_1,...,x_{n+1})$ of congruences of the form $Q(x_1,...,x_{n+1}) \equiv 0 \bmod q$ with $q = p^n$, where $\max\{|x - x_{0_1}|,...,|x_{n+1} - x_{0_{n+1}}|\} \leq N$ and $(x_n^2 - x_{n+1}^2, p) = 1$. In fact, we consider a smooth version of this problem and establish an asymptotic formula when $n \to \infty$, under the condition $N \geq q^{\frac{1}{2}+\varepsilon}$.

THE EISENSTEIN HOMOLOGY AND COHOMOLOGY GROUPS FOR IMAGINARY QUADRATIC FIELDS

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Let $\mathcal{O}$ be the ring of integers of an imaginary quadratic field K and Γ be a subgroup of finite index of the full Bianchi modular group $G = \mathrm{SL}_2(\mathcal{O})$. Let $M_k(\Gamma)$ be the complex vector space of modular forms of scalar weight k for the subgroup Γ with $S_k(\Gamma)$ be the subspace of cusp forms. There is a natural complement $E_k(\Gamma)$ of $S_k(\Gamma)$ inside $M_k(\Gamma)$, Eisenstein subspace of $M_k(\Gamma)$. To study arithmetic properties like explicit Fourier coefficients of modular forms in $M_k(\Gamma)$, it is natural to study the dual space, the space of modular symbols $\mathcal{M}_k(\Gamma)$ (basically the homology groups or the dual cohomology groups of the modular curve associated to Γ). Denote the corresponding space of the cuspidal and Eisenstein part by $\mathcal{S}_k(\Gamma)$ and $\mathcal{E}_k(\Gamma)$. The aim of this article is to study the space $\mathcal{E}_k(\Gamma)$.

Eisenstein cycles also known as Eisenstein elements, are elements in the Eisenstein homology groups of Γ. Merel invented Eisenstein cycles in order to give an expression for elements in the Eisenstein ideals [8] and have recently been applied to the study of the rank of Eisenstein ideals of Mazur (cf. [6], [7]). These Eisenstein cycles are determined by *periods* of the Eisenstein series. Much remains to be known about extensions of these results to the Bianchi modular forms.

Banerjee-Merel [1] studied the Eisenstein cycles as modular symbols for the congruence subgroups in the classical case of elliptic modular forms. They write down the Eisenstein cycles in the first homology group of modular curves of level N as $\overline{\mathbb{Q}}$ linear combinations of Manin symbols for any odd integer N. They generate the Eisenstein part of the space of modular symbols. In [2], they extended the result to the subgroups of finite indices of the full modular group. Of course, in general these Eisenstein cycles are not necessarily $\overline{\mathbb{Q}}$ valued. In continuation, we extend the computation to the Bianchi modular forms. In our setting, differential forms are harmonic rather than holomorphic.

John Cremona and his collaborators initiated the study of modular symbols (homology groups of quotients of three-dimensional hyperbolic space) for the imaginary quadratic fields. They are mostly interested in the cuspidal part of the homology group. Over imaginary quadratic fields, Ito [5] showed that the periods are again described by Sczech's Dedekind sums [9] for $\Gamma = \mathrm{SL}_2(\mathcal{O})$. In the case of congruence subgroups, however the analog of elliptic Dedekind sums are defined yet.

The study of Eisenstein part of the cohomology started with the pioneer work of [4]. In [3], Sengün and his co-authors studied the dimension of the cohomology of Bianchi groups using Sczech cocycles for the *principal* congruence subgroups. It is natural to ask if the computation can be generalized to other important congruence subgroups like $\Gamma_0(N)$ or $\Gamma_1(N)$.

In our work, we extend the study of Eisenstein elements to subgroups of the Bianchi groups of the form $\Gamma_1(N)$, We use Sczech cocycles [10] to study the Eisenstein part of the cohomology. Note that Sczech cocycles determine the periods of the Eisenstein series. The main novelty of our work for the subgroup $\Gamma_1(N)$ stems from the explicit understanding of cusps for these subgroups. We also give reasons why the same computation may not be extended for the subgroups of the form $\Gamma_0(N)$. Although, we understand the cuspidal subgroup for $\Gamma_0(N)$.

References

[1] D. Banerjee and L. Merel, *The Eisenstein cycles as modular symbols*, J. Lond. Math. Soc. (2) **98** (2018), no. 2, 329–348.
[2] ———, *The Eisenstein cycles and Manin Drinfeld properties*, arXiv preprint arXiv:2204.06379 (2022)
[3] M. H. Şengün and S. Türkelli, *Lower bounds on the dimension of the cohomology of Bianchi groups via Sczech cocycles*, J. Théor. Nombres Bordeaux **28** (2016), no. 1, 237–260.
[4] G. Harder, *Eisenstein cohomology of arithmetic groups. The case* GL_2, Invent. Math. **89** (1987), no. 1, 37–118.
[5] H. Ito, *A function on the upper half space which is analogous to the imaginary part of* $\log \eta(z)$, J. Reine Angew. Math. **373** (1987) 148–165.
[6] E. Lecouturier, *Higher Eisenstein elements, higher Eichler formulas and rank of Hecke algebras*, Invent. Math. **223** (2021), no. 2, 485–595.
[7] B. Mazur, *Modular curves and the Eisenstein ideal*, Inst. Hautes Études Sci. Publ. Math. (1977), no. 47, 33–186 (1978).
[8] L. Merel, *L'accouplement de Weil entre le sous-groupe de Shimura et le sous-groupe cuspidal de* $J_0(p)$, J. Reine Angew. Math. **477** (1996) 71–115.
[9] R. Sczech, *Dedekindsummen mit elliptischen Funktionen*, Invent. Math. **76** (1984), no. 3, 523–551.
[10] ———, *Dedekind sums and power residue symbols*, Compositio Math. **59** (1986), no. 1, 89–112.

IDENTITIES FOR THE ROGERS–RAMANUJAN CONTINUED FRACTION AND FUNCTIONS

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For $|q| < 1$, the well-known Rogers–Ramanujan continued fraction $R(q)$ is defined by $R(q) := \frac{q^{1/5}}{1} \underset{+}{} \frac{q}{1} \underset{+}{} \frac{q^2}{1} \underset{+}{} \frac{q^3}{1} \underset{+}{} \cdots$, whereas the Rogers–Ramanujan functions $G(q)$ and $H(q)$ are defined by

$$G(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} \quad \text{and} \quad H(q) := \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q;q)_n},$$

where $(a;q)_n := \prod_{k=0}^{n-1}(1 - aq^k)$. In this paper, we prove some new modular identities for the Rogers–Ramanujan continued fraction. For example, we prove that

$$R(q)R(q^4) = \frac{R(q^5) + R(q^{20}) - R(q^5)R(q^{20})}{1 + R(q^5) + R(q^{20})},$$

$$\frac{1}{R(q^2)R(q^3)} + R(q^2)R(q^3) = 1 + \frac{R(q)}{R(q^6)} + \frac{R(q^6)}{R(q)},$$

and

$$R(q^2) = \frac{R(q)R(q^3)}{R(q^6)} \cdot \frac{R(q)R^2(q^3)R(q^6) + 2R(q^6)R(q^{12}) + R(q)R(q^3)R^2(q^{12})}{R(q^3)R(q^6) + 2R(q)R^2(q^3)R(q^{12}) + R^2(q^{12})}.$$

In the process, we also find some new relations for the Rogers–Ramanujan functions by using dissections of theta functions and the quintuple product identity. For example, we prove that

$$\frac{G\left(q^2\right) G\left(q^3\right)}{H\left(q\right) G\left(q^6\right)} - \frac{H\left(q^2\right) H\left(q^3\right)}{G\left(q\right) H\left(q^6\right)} = q\frac{(q^5;q^5)_\infty^2(q^{30};q^{30})_\infty^2}{(q^{10};q^{10})_\infty^2(q^{15};q^{15})_\infty^2}$$

and

$$\frac{H\left(q\right) G\left(q^6\right)}{H\left(q^2\right) H\left(q^3\right)} - q^2\frac{G\left(q\right) H\left(q^6\right)}{G\left(q^2\right) G\left(q^3\right)} = \frac{(q^{10};q^{10})_\infty^2(q^{15};q^{15})_\infty^2}{(q^5;q^5)_\infty^2(q^{30};q^{30})_\infty^2}.$$

EXPLICIT ZERO REPULSION OF DIRICHLET L-FUNCTIONS

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The potential existence of Landau–Siegel zeros creates significant challenges to establishing estimates on the distribution of primes in arithmetic progressions. Zero repulsion is one of the fundamental tools for overcoming these challenges. There are numerous variations of zero repulsion estimates, many of which focus on low-lying zeros.

Using elegant sieve methods, Graham established the current best known zero repulsion estimate that holds uniformly for any height and any modulus. By employing additional sophisticated optimization techniques, Heath-Brown improved this result for large moduli and low-lying zeros. However, neither of their works are completely explicit.

We establish an explicit Deuring-Heilbronn zero repulsion phenomenon for Dirichlet L-functions modulo q. Our estimate is uniform in the entire critical strip and improves over the previous explicit estimate due to Thorner and Zaman.

MODULAR IDENTITIES ON SPECIAL CASES OF RAMANUJAN'S GENERAL CONTINUED FRACTION

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In this article, we introduce some continued fractions $K(q), L(q)$ and $M(q)$ based on Ramanujan-Selberg continued fraction $C(q)$. Then we obtain identities connecting $K(q)$ with $K(-q)$, $K(q^i)$, $i = 2, 3, 5, 7$, $L(q)$ with $L(-q)$, $L(q^j)$, $M(q)$ with $M(-q)$, $M(q^j)$, $j = 2, 3$, and $C(q)$ with $C(-q)$. Also, we fetch some more algebraic results on $K(q)$ and $L(q)$. In addition to this, we deduce few fascinating colored partitions as an application which also validate the results.

Key words: Continued fraction, Modular equations, Theta functions.
MSC Classification: 11F17, 11F27, 23C05.

PARITY DISTRIBUTION AND DIVISIBILITY OF MEX-RELATED PARTITION FUNCTIONS

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Andrews and Newman introduced the mex-function $mex_{A,a}(\lambda)$ for an integer partition λ of a positive integer n as the smallest positive integer congruent to a modulo A that is not a part of λ. They then defined $p_{A,a}(n)$ to be the number of partitions λ of n satisfying $mex_{A,a}(\lambda) \equiv a \pmod{2A}$.They found the generating function for $p_{t,t}(n)$ and $p_{2t,t}(n)$ for any positive integer t, and studied their arithmetic properties for some small values of t. In our article, we study the partition function $p_{mt,t}(n)$ for all positive integers m and t. We show that for sufficiently large X, the number of all positive integers $n \leq X$ such that $p_{mt,t}(n)$ is an even number is at least $O(\sqrt{X/3})$ for all positive integers m and t . We also prove that for sufficiently large X, the number of all positive integers $n \leq X$ such that $p_{mp,p}(n)$ is an odd number is at least $O(loglogX)$ for all $m \not\equiv 0 \pmod 3$ and all primes $p \equiv 1 \pmod 3$. Finally, we establish identities connecting the ordinary partition function to $p_{mt,t}(n)$.

SUM OF THE TRIPLE DIVISOR FUNCTION AND FOURIER COEFFICIENTS OF $SL(3, \mathbb{Z})$ HECKE-MAASS FORMS OVER QUADRATIC POLYNOMIAL

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The quest for an asymptotic formula that provides highly accurate approximations for the averages of arithmetical functions lies at the core of numerous central problems in number theory. For instance, the sum $\sum_{n \leq X} \mathcal{A}(p(n))$, where $\mathcal{A}(n)$ be an arithmetical function and $p(n)$ be a polynomial over $\mathbb{Z}$, has been widely studied. The literature has shown a sustained interest in examining the sizes and oscillations of the Dirichlet coefficients of $GL(m)$ L-functions. Calculating averages over such sparse sequences finds applications in analysing moments of L-functions and in establishing non-vanishing results. The fundamental objective within the theory of automorphic forms is to assess sums comprising Hecke eigenvalues. In this talk, we will establish an asymptotic estimate for

$$\sum_{1 \leq m_1, m_2 \leq X} \mathcal{A}(p(m_1, m_2)),$$

where $\mathcal{A}(n)$ is either the Fourier coefficients of $SL(3, \mathbb{Z})$ Hecke-Maass cusp forms i.e., $A(1, n)$ or the triple divisor function $d_3(n) := \{(n_1, n_2, n_3) \in \mathbb{Z}_+^3 : n = n_1 n_2 n_3\}$ and $p(x, y) \in \mathbb{Z}[x, y]$ is a quadratic polynomial.

EXTREME VALUES OF L-FUNCTIONS AT CRITICAL POINTS OF THE ZETA FUNCTION

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We compute extreme values of Dirichlet L- functions at the critical points of the zeta function to the right of $\Re s = 1$. That is, we estimate large and small values of $|L(\rho', \chi)|$, where χ is a primitive character mod q for $q > 2$ and ρ' runs over critical points of the Riemann zeta function in the right half of the one-line and on the one-line, that is, the points where $\zeta'(\rho') = 0$ and $1 \leq \Re\rho'$.

It shows statistical independence of Dirichlet L- functions and the Riemann zeta function in a certain way as these values are very similar to the values taken by Dirichlet L-functions without any restriction.

CONGRUENCES MODULO POWERS OF 2 FOR RESTRICTED PARTITION TRIPLES DUE TO LIN AND WANG

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In 2018, Lin and Wang (*Colloq. Math.* 2018) introduced two restricted partition triples, whose generating functions are related to the reciprocals of Ramanujan–Gordon identities. Let $RG_2(n)$ denote the number of partitions triples of n, where odd parts in the first two components are distinct and the last component only contains even parts. Lin and Wang proved some congruences modulo 5 and 7 satisfied by $RG_2(n)$. They further asked the existence of the infinite families of congruences modulo arbitrary powers of 2 for the function $RG_2(n)$. Utilizing some q-techniques, we prove that such congruence families indeed exist.

INHOMOGENEOUS KHINTCHINE-GROSHEV TYPE THEOREMS ON MANIFOLDS OVER FUNCTION FIELDS

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We talk about a complete Khintchine-Groshev type theorem, which is one of the most fundamental results in metric theory of Diophantine approximation, in both homogeneous and inhomogeneous settings, on analytic nondegenerate manifolds over a local field of positive characteristic. We apply a dynamical technique, originally invented by D.Y. Kleinbock and G.A. Margulis, in order to prove homogeneous convergence case. The inhomogeneous version is then proved using the Inhomogeneous Transference Principle and techniques of Ubiquitous Systems.

MODULAR RELATIONS INVOLVING GENERALIZED DIGAMMA FUNCTIONS

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Generalized digamma functions $\psi_k(x)$, studied by Ramanujan, Deninger, Dilcher, Kanemitsu, Ishibashi etc., appear in the Laurent series coefficients of the Hurwitz zeta function. In this paper, a *modular relation* of the form $F_k(\alpha) = F_k(1/\alpha)$ containing the infinite series of $\psi_k(x)$, or, equivalently, between the generalized Stieltjes constants $\gamma_k(x)$, is obtained for any $k \in \mathbb{N}$. When $k = 0$, it reduces to the following famous transformation given on page 220 of Ramanujan's Lost Notebook,

$$\sqrt{\alpha}\left\{\frac{\gamma - \log(2\pi\alpha)}{2\alpha} + \sum_{n=1}^{\infty}\phi(n\alpha)\right\} = \sqrt{\beta}\left\{\frac{\gamma - \log(2\pi\beta)}{2\beta} + \sum_{n=1}^{\infty}\phi(n\beta)\right\}$$
$$= -\frac{1}{\pi^{3/2}}\int_0^\infty \left|\Xi\left(\frac{1}{2}t\right)\Gamma\left(\frac{-1+it}{4}\right)\right|^2 \frac{\cos\left(\frac{1}{2}t\log\alpha\right)}{1+t^2}\,dt,$$

where

$$\phi(x) := \psi(x) + \frac{1}{2x} - \log(x),$$

and $\Xi(t)$ is the Riemann Ξ-function.

For $k = 1$, an integral containing $\Xi(t)$ corresponding to the aforementioned modular relation, is also obtained along with its asymptotic expansions as $\alpha \to 0$ and $\alpha \to \infty$. Carlitz-type and Guinand-type finite modular relations involving $\psi_j^{(m)}(x), 0 \leq j \leq k, m \in \mathbb{N}\cup\{0\}$, are also derived, thereby extending previous results on the digamma function $\psi(x)$. The above mentioned modular relations involving the infinite series of $\psi_k(x)$ can also be obtained from these finite modular relations, by subtracting the necessary terms and tending a suitable parameter to ∞. The Guinind-type result involving finite sum of $\psi_j^m(x)$ also further gives 2-parameter analogues of the well known duplication formula of $\psi(x)$.

The extension of Guinand's result for $\psi_j^{(m)}(x), m \geq 2$, involves an interesting combinatorial sum $h(r)$ over integer partitions of $2r$ into exactly r parts. This sum plays a crucial role in an inversion formula needed for this extension. This formula, having connection with the inversion formula for the inverse of a triangular Toeplitz matrix, is a independent result and can be used with other arithmetic functions. The modular relation for $\psi_j'(x)$ is subtle and requires delicate analysis. It is indeed very interesting to see all the identities established carry the exact symmetrical nature we began with.

EQUIVALENT CRITERIA FOR THE RIEMANN HYPOTHESIS FOR CHANDRASEKHARAN-NARASIMHAN CLASS OF L-FUNCTIONS

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A noteworthy equivalent criteria, attributed to Riesz in 1916 has shown that the Riemann hypothesis is equivalent to the following bound for an infinite series associated with $\mu(n)$,

$$P_2(x) := \sum_{n=1}^{\infty} \frac{\mu(n)}{n^2} \exp\left(-\frac{x}{n^2}\right) = O_\epsilon\left(x^{-\frac{3}{4}+\epsilon}\right), \quad \text{as } x \to \infty, \tag{1}$$

for any positive ϵ. Driven by this motivation, Hardy and Littlewood established another equivalent criterion for the Riemann hypothesis while rectifying an identity of Ramanujan. Namely, they showed that

$$P_1(x) := \sum_{n=1}^{\infty} \frac{\mu(n)}{n} \exp\left(-\frac{x}{n^2}\right) = O_\epsilon\left(x^{-\frac{1}{4}+\epsilon}\right), \quad \text{as } x \to \infty. \tag{2}$$

In this talk, we will explore a generalization of Ramanujan-Hardy-Littlewood identity for Chandrasekharan-Narasimhan class of L-functions with a certain twist satisfying functional equation with a single factor of $\Gamma(As+B)$. We will delve into several particular instances including holomorphic Hecke eigenform of weight ω. We will also discuss equivalent criteria for the Riemann hypothesis for the L-functions falling within this category. Furthermore, we will unveil an entirely novel form of equivalent criteria for the Riemann hypothesis of the Riemann zeta function $\zeta(s)$.

Hybrid Subconvexity bound for $GL(3) \times GL(2)$ L-functions: t and level aspect

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Abstract

Studies related to the analytic behavior of L-functions are the focus of research in modern Analytic Number Theory due to their applications in many prominent problems of Number Theory and other mathematics. One of the important analytic properties of L-functions is knowing their growth inside the critical strip, in terms of the parameter involved in the conductor of L-function. By using some standard tools, we can give an upper bound for an L-function which is one-fourth power of the corresponding conductor and is called the convexity bound. The subconvexity problem for a given L-function asks for a sharper upper bound than the convexity bound.

The subconvexity problem has very rich literature for the different L-functions. Let π be any Hecke-Maass cusp form for $SL(3, \mathbb{Z})$ and f be any holomorphic cusp form for the congruent subgroup $\Gamma_0(M)$ of $SL(2, \mathbb{Z})$, let the associated degree-six Rankin-Selberg L-functions is $L(\sigma + it, \pi \times f)$. We will prove the subconvexity for $L(1/2 + it, \pi \times f)$ in terms of the level of $GL(2)$ form M and t, where $M = p_1 p_2$, which are primes with $p_1 < p_2$. Here our conductor for $L(1/2 + it, \pi \times f)$ will be $\mathcal{Q} = M^3(1 + |t|)^6$.

This is a joint work with my Ph.D. thesis advisor Prof. Saurabh Kumar Singh and one collaborator Dr. Sumit Kumar. In this talk, Firstly, I will give an overview of some past research that has been done in the direction of our result. Then I will properly state our result and explain why it is interesting. If time permits, I will briefly explain the key ideas involved in the proof.

ON TERMS IN A DYNAMICAL DIVISIBILITY SEQUENCE HAVING A FIXED G.C.D WITH THEIR INDICES

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Let F be an integer polynomial with degree at least 2. Define the sequence (a_n) by $a_n = F(a_{n-1})$ for $n \geq 1$ and $a_0 = 0$. For each positive integer k, let $\mathcal{A}_{F,k}$ be the set of positive integers n such that $\gcd(n, a_n) = k$. We prove that for almost all polynomials F, the asymptotic density of $\mathcal{A}_{F,k}$ exists and also give an effective criterion to check if the density is positive.

TRANSFORMATIONS FOR APPELL SERIES OVER FINITE FIELDS AND TRACES OF FROBENIUS FOR ELLIPTIC CURVES

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Using properties of certain character sums over finite fields, we here obtain some transformations for finite field Appell series F_2^* and F_4^* in terms of ${}_4F_3$-Gaussian hypergeometric series. These motivate us to find a finite field analog for certain transformations satisfied by the classical Appell series F_2 and F_4. We also deduce an expression for the finite field Appell series F_4^* in terms of ${}_3F_2$-Gaussian hypergeometric series. Consequently, we investigate connections of trace of Frobenius endomorphism for certain families of elliptic curves and the finite field Appell series F_4^*.

STABILITY OF 2-CLASS GROUPS IN $\mathbb{Z}_2$-EXTENSION OF CERTAIN REAL QUADRATIC NUMBER FIELDS

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Let K be a number field and p be a prime number, and K_∞ be a $\mathbb{Z}_p$-extension of K. For $n \geq 0$, let K_n be the corresponding n^{th}-layer of K_∞. Let X_∞ denote the the inverse limit (over n) of the p-class group A_n of K_n with respect to the norm map. Iwasawa proved that for sufficiently large n, there exist constants λ, μ and ν such that $\#A_n = p^{\mu p^n + \lambda n + \nu}$. Later, Greenberg conjectured that in case of the cycolotomic $\mathbb{Z}_p$-extension of a totally real number field, $\lambda = \mu = 0$. For a real quadratic field $K = \mathbb{Q}(\sqrt{d})$ with d having three distinct prime factors, it has been proven that under certain conditions, the Iwasawa module X_∞ corresponding to the cyclotomic $\mathbb{Z}_2$-extension of K is cyclic. In this talk, we shall see that with some elementary assumptions on the prime factors of d, X_∞ is isomorphic to $\mathbb{Z}/2\mathbb{Z}$. Consequently, we prove that the Iwasawa invariants λ and μ for such fields are equal to 0, validating Greenberg's conjecture for these fields.

A SERIES ASSOCIATED TO RANKIN-SELBERG L-FUNCTION AND MODIFIED K-BESSEL FUNCTION

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The Ramanujan cusp form $\Delta(z)$ of weight 12 over the full modular group $SL_2(\mathbb{Z})$ is defined by $\Delta(z) := e^{2\pi i z}\prod_{n=1}^{\infty}\left(1-e^{2\pi i n z}\right)^{24} = \sum_{n=1}^{\infty}\tau(n)e^{2\pi i n z}$, and $\tau(n)$ denotes the nth Fourier coefficient, which is also popularly known as the Ramanujan tau function. Don Zagier predicted that the constant term of the automorphic function $\mathcal{F}(z) = y^{12}|\Delta(z)|^2$, with $\Im(z) = y$, that is, the infinite series $a_0(y) := y^{12}\sum_{n=1}^{\infty}\tau^2(n)e^{-4\pi n y}$, has a asymptotic expansion involving non-trivial zeros of $\zeta(s)$. In 2000, Hafner and Stopple showed the above asymptotic behaviour of $a_0(y)$ as $y \to 0^+$ assuming the Riemann hypothesis. In a recent work, Berndt, Dixit, Gupta, and Zaharescu showed that, for $\Re(\nu), \Re(y) > 0$, we have

$$\sum_{n=1}^{\infty}\tau(n)n^{\nu/2}K_\nu(y\sqrt{n}) = 2^{35+\nu}y^{\nu}\pi^{12}\Gamma(12+\nu)\sum_{n=1}^{\infty}\frac{\tau(n)}{(y^2+16\pi^2 n)^{\nu+12}},$$

where K_ν denotes the modified K-Bessel function of 2nd kind of order ν. In this talk, we will present exact formulae for the series $\sum_{n=1}^{\infty}c_f^{\ell}(n)n^{\nu/2}K_\nu\left(y\sqrt{n}\right)$, where $\ell \in \{1,2\}$ and $c_f(n)$ denotes the nth Fourier coefficient of a Hecke eigenform $f(z)$ over $SL_2(\mathbb{Z})$. The aforementioned series corresponding to $\ell = 1$ enabled us to generalize an identity of Berndt et al. For $\ell = 2$, we have obtained an exact formula as well as asymptotic expansion of the series as $y \to 0^+$ and also observe that it has a connection with the non-trivial zeros of $\zeta(s)$ analogues to the conjecture of Zagier.

WIENER-IKEHARA TAUBERIAN THEOREM WITH IMPROVED ERROR

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The Wiener-Ikehara theorem is an important theorem in Tauberian theory, which gives an asymptotic estimate for the sum of the coefficients of a Dirichlet series $\sum_{n=1}^{\infty} \frac{a_n}{n^s}$. This theorem has many applications in analytic number theory; for example, it provides a simpler proof of the Prime Number Theorem. In this talk, we present an elegant proof of the Wiener-Ikehara Tauberian theorem which relies only on basic Fourier analysis and known estimates for the given Dirichlet series. Using this technique, we also give a version of the Wiener-Ikehara theorem with an error term.

ON INEQUALITIES RELATED WITH A GENERALIZED EULER TOTIENT FUNCTION AND LUCAS SEQUENCES

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Let $\varphi(n)$ be the Euler totient function of n, defined as the number of positive integers less than or equal to n that are co-prime with n. In this paper, we consider the function φ_k, a generalization of φ, and establish some inequalities related to Lucas sequences of the first kind $(U_n)_{n\geq 1}$ with characteristic equation having real roots. As an application to these inequalities, we further establish inequalities related to Fibonacci, Pell, and balancing sequences.

THETA-FUNCTION IDENTITIES, EXPLICIT VALUES FOR RAMANUJAN'S CONTINUED FRACTIONS OF ORDER SIXTEEN AND APPLICATIONS TO PARTITION THEORY

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Two continued fractions of order sixteen connected with the Ramanujan-Göllnitz-Gordon continued fraction are derived from a general continued fraction identity of Ramanujan. Theta-function identities and general theorems for the explicit evaluations of the two continued fractions are established. Some colour partition identities and matching coefficient results are also derived as applications to theta-function identities.

DEFICIENT AND NEAR F_K-PERFECT NUMBERS

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For a positive integer n, let the arithmetic function $\sigma_2(n)$ denote the sum of squares of all the positive divisors of n. A positive integer n is called an F-perfect number if the sum of the squares of all the proper divisors of n is three times the number, i.e. $\sigma_2(n)-n^2 = 3n$. This paper introduces two new type of numbers, near F-perfect number and deficient F-perfect numbers. A positive integer n, called a near F-perfect number if $\sigma_2(n) - n^2 - d^2 = 3n$. Similarly, n is known as deficient F-perfect number if $\sigma_2(n) - n^2 + d^2 = 3n$. Here d is the redundant divisor of n. We discuss a few characterisations of these new type of numbers and develop the restrictions in order to bound these type of numbers. Also, this research works tries to generalise these numbers and establish a relationship with the F-perfect numbers.

ON K-FACILE PERFECT NUMBERS

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Let $\sigma(n)$ be the sum of divisors of a positive integer n; n is abundant if $\sigma(n) > 2n$. An abundant number n is said to be a k-facile perfect number if

$$\sigma(n) = 2n + d_1 d_2 d_3 \cdots d_k$$

where d_i's, $1 \leq i \leq k$ are distinct divisors of n that are greater than one called facile divisors. In this paper, we characterise k- perfect numbers and establish a relationship between k-facile perfect numbers and other unique numbers.

MONOGENITY OF ITERATES OF POLYNOMIALS

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Let K be a field. A polynomial $f(x) \in K[x]$ is called stable if all of its iterates are irreducible over K. We say that a number field K is monogenic if there exists α in $\mathcal{O}_K$, the ring of integers of K, such that $\mathcal{O}_K = \mathbb{Z}[\alpha]$. In this talk, we discuss the monogenity of a tower of number fields defined by the iterates of a stable polynomial. We give a necessary condition for the monogenity of the number fields defined by the iterates of a stable polynomial. When the stable polynomial is of certain type, we also give a sufficient condition for the monogenity of the fields defined by each of its iterate. As a consequence, we obtain an infinite 3-tower of monogenic number fields. Moreover, we construct an infinite family of stable polynomials such that each of its iterate is non-monogenic.

EULER PRODUCT ASYMPTOTICS FOR L-FUNCTIONS OF ELLIPTIC CURVES

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During his study of highly composite numbers, Ramanujan established an explicit formula that describes the asymptotic behaviour of the partial Euler product of the Riemann zeta function in right half of the critical strip $\frac{1}{2} < \mathrm{Re}(s) < 1$. In this talk, we will explain current work in progress where we prove an analogous formula for partial Euler products of L-functions attached to elliptic curves over $\mathbb{Q}$.

As an application of our result, we give a new proof of Goldfeld's theorem that the original version of the Birch and Swinnerton-Dyer conjecture implies the modern formulation of the conjecture: we prove that if $E/\mathbb{Q}$ is an elliptic curve with rank r, and if N_p denotes the number of points of E modulo each prime p, then the asymptotic $\prod_{p \leq x} \frac{N_p}{p} \sim C(\log x)^r$ as $x \to \infty$ implies that $\mathrm{ord}_{s=1} L(E,s) = r$.

CERTAIN DIOPHANTINE EQUATIONS AND NEW PARITY RESULTS FOR 21-REGULAR PARTITIONS

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For a positive integer $t \geq 2$, let $b_t(n)$ denote the number of t-regular partitions of a nonnegative integer n. In a recent paper, Keith and Zanello investigated the parity of $b_t(n)$ when $t \leq 28$. They discovered new infinite families of Ramanujan type congruences modulo 2 for $b_{21}(n)$ involving every prime p with $p \equiv 13, 17, 19, 23 \pmod{24}$. In this paper, we investigate the parity of $b_{21}(n)$ involving the primes p with $p \equiv 1, 5, 7, 11 \pmod{24}$. We prove new infinite families of Ramanujan type congruences modulo 2 for $b_{21}(n)$ involving the odd primes p for which the Diophantine equation $8x^2 + 27y^2 = jp$ has primitive solutions for some $j \in \{1, 4, 8\}$, and we also prove that the Dirichlet density of such primes is equal to $1/6$.

Number of $\mathbb{F}_q$-points on Diagonal hypersurfaces and hypergeometric function

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Abstract

Let D_λ^d denote the family of monomial deformations of diagonal hypersurface over a finite field $\mathbb{F}_q$ given by

$$D_\lambda^d : X_1^d + X_2^d + \cdots + X_n^d = \lambda d X_1^{h_1} X_2^{h_2} \cdots X_n^{h_n},$$

where $d, n \geq 2$, $h_i \geq 1$, $\sum_{i=1}^n h_i = d$, and $\gcd(d, h_1, h_2, \ldots, h_n) = 1$. The Dwork hypersurface is the case when $d = n$, that is, $h_1 = h_2 = \cdots = h_n = 1$. Formulas for the number of $\mathbb{F}_q$-points on the Dwork hypersurfaces in terms of McCarthy's p-adic hypergeometric functions are known. In this talk we see a formula for the number of $\mathbb{F}_q$-points on D_λ^d in terms of McCarthy's p-adic hypergeometric function which holds for $d \geq n$.

A CONGRUENCE IDENTITY ON ORDERED PARTITION USING PERMUTATION POLYNOMIALS

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In this paper, we have investigated a class of permutation polynomials of the form $h_k^t(x) = \sum_{n=1}^{k} n^t x^n$ over a finite field F_p, where k and t are positive integers, and p is an odd prime. We find certain conditions on k and t for $h_k^t(x)$ to be a permutation polynomial over F_p. We then use this class of permutation polynomial to find a new congruence identity on the parts of ordered partitions of $(p-1), 2(p-1), \dots, \mu(p-1)$ into s parts, where μ is the largest integer satisfying $\mu(p-1) \leq ks$.