DIVERGENCE FREE ENTROPY STABLE SCHEMES FOR TWO-FLUID RELATIVISTICS PLASMA FLOW EQUATIONS

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This study presents a constraint-preserving entropy-stable method for the two-fluid relativistic plasma flow equations. The fundamental equation structure involves the collective interaction of two charged fluids with the complete set of Maxwell's equations. Existing Numerical methods commonly overlook or approximate the divergence-free conditions for the magnetic field and the divergence constraint for the electric field by utilizing the hyperbolic form of Maxwell's equations. However, this approach proves to be inadequate, particularly in scenarios involving periodic boundary conditions. Another less commonly employed technique involves the staggered evolution of electromagnetic quantities, but its implementation is challenging.

In this study, we propose a second-order constraint-preserving entropy stable scheme that does not necessitate a staggered mesh. This makes the proposed scheme simple and can be easily implemented in any finite volume or finite difference code. It utilizes a multidimensional Riemann solver-based treatment of electromagnetic quantities at each cell corner to construct suitable fluxes across edges. The scheme ensures divergence-free evolution of the magnetic field and preserves the electric field's divergence constraint up to the discretization error. Combined with entropy stable schemes for the fluid part, it results in a provably stable numerical method.

Finally we design SSP-RK schemes to evolve the solution in time and ARK-IMEX schemes so that they provide a strong coupling between the both finite-volume based fluid solver and the electromagnetic fields.

To evaluate the proposed methods' computational performance, various challenging test problems are simulated. The results demonstrate that the scheme ensures divergence-free evolution of the magnetic field up to the machine precision and preserves the electric field's divergence constraint, accounting for discretization errors.

HIGHER ORDER MIXED FINITE ELEMENTS FOR MAXWELL'S EQUATIONS

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Computational electromagnetics entails numerical solution of Maxwell's equations and has been one of the foundational pillars of modern electrical engineering. In this work, we demonstrate higher order, structure preserving finite element methods for the following system of Maxwell's equations:

$$\begin{aligned} \frac{\partial p}{\partial t} + \nabla E &= f_p \text{ in } \Omega \times (0,T], \\ \nabla p + \frac{\partial E}{\partial t} - \nabla \times H &= f_E \text{ in } \Omega \times (0,T], \\ \frac{\partial H}{\partial t} + \nabla \times E &= f_H \text{ in } \Omega \times (0,T], \end{aligned}$$

where $\Omega \subset \mathbb{R}^2/\mathbb{R}^3$ is a domain with Lipschitz boundary $\partial\Omega$ and with the following homogeneous boundary conditions: $p = 0, E \times n = 0, H \cdot n = 0$ on $\partial\Omega \times (0,T]$, where n is the unit outward normal to $\partial\Omega$, and with the following initial conditions: $p(x,0) = p_0(x), E(x,0) = E_0(x),$ and $H(x,0) = H_0(x)$ for $x \in \Omega$. We shall characterize the solution of this problem posed using a mixed variational formulation as follows.

Theorem 1 (Well Posedness)**.** *Let $f_p \in L^1[0,T] \times L^2(\Omega)$, $f_E \in L^1[0,T] \times L^2(\Omega)$, and $f_H \in L^1[0,T] \times L^2(\Omega)$. Then the solution (p,E,H) of the Maxwell's equations posed using a mixed variational formulation with the given initial and boundary conditions and with sufficient regularity satisfies:*

$$\|p\| + \|E\| + \|H\| \leq C\Big[\|p_0\| + \|E_0\| + \|H_0\| + \|f_p\| + \|f_E\| + \|f_H\|\Big],$$

for a positive bounded constant C and appropriate choices of the norms.

We shall then demonstrate computational results for some model problems in two and three dimensions using backward Euler and Crank-Nicholson schemes for the time discretization and finite elements for the spatial discretization. Our finite elements spaces shall be drawn from a de Rham sequence of conforming finite dimensional polynomial function spaces spanned by linear and quadratic Lagrange polynomial, and Nédélec and Raviart-Thomas vector basis elements.

A NEW SPACE TRANSFORMED FINITE ELEMENT METHOD (ST-FEM) FOR ELLIPTIC PROBLEMS IN R^N WITH SPHERICAL INTERFACES

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Interface problems, where distinct materials or physical domains meet, present appalling challenges in numerical simulations due to the discontinuities and sharp gradients across interfaces. Traditional finite element methods struggle to capture such behavior accurately. A new space transformed finite element method (ST-FEM) is developed for solving elliptic interface problems in R^n. A homeomorphic stretching transformation is employed to obtain an equivalent problem in the transformed domain which can be solved easily, and the solution can be projected back to original domain by the inverse transformation. Compared with the existing methods, this new scheme has capability of handling discontinuities across the interface. The proposed approach has advantages in circumventing interface approximation properties and reducing the degree of freedom. We first develop ST-FEM for elliptic problems and then extend the idea to treat elliptic interface problems. We prove optimal a priori error estimates in the H^1 and L^2 norms, and quasi-optimal error estimate for the maximum norm. Finally, numerical experiments demonstrate the superior accuracy and convergence properties of the ST-FEM when compared to the standard finite element method. The interface is assumed to be a $(n-1)$-sphere, however, our analysis can cover symmetric domains like an ellipsoid or a cylinder.

A FOURTH-ORDER ACCURATE NUMERICAL METHOD FOR BLACK-SCHOLES PDES ARISING IN OPTION PRICING

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We consider generalized Black-Scholes equation for pricing various type of options. Since spatial domain in Black-Scholes PDE is unbounded so first we need to bound the spatial domain. We truncate our spatial domain to some sufficiently chosen large number. After that we used Crank-Nicolson scheme to discretize time variable and standard central difference scheme to discretize spatial variable. This scheme gives us second-order accuracy in both space and time. In order to enhance the order of convergence of the proposed scheme, we apply the Richardson extrapolation technique, which gives fourth-order accuracy in both variables. The stability and convergence are studied. To validate the proposed method, several numerical experiments are carried out.

ON POLYNOMIAL TWO PARAMETER EIGENVALUE PROBLEM

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In this talk, a Homotopy based numerical method will be presented to find the numerical solution of polynomial two parameter eigenvalue problems ($\mathbb{PTEP}$). In finding the numerical solution, generally $\mathbb{PTEP}$ is converted into its associated linear two-parameter eigenavlue $\mathbb{LTEP}$ using linearization techniques, and then the problem is solved by converting the $\mathbb{LTEP}$ into the usual joint generalised eigenavlue problem ($\mathbb{GEP}$) with Kronecker product type coefficient matrices of high dimension. But the Homotopy method does not require to convert $\mathbb{PTEP}$ into its joint $\mathbb{GEP}$ and thus it reduces computational cost and memory storage. Again, to track the homotopy path, we will use Euler method for finding the predictor and Inverse Iteration for it's correction.

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Least square method for wave interaction with a tunnel and a submerged porous plate over a trench-type bottom

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The boundary value problem (bvp) involving oblique wave interaction with a tunnel in the presence of a submerged horizontal porous plate over a trench-type bottom is examined for its solution. The problem is formulated based on Darcy's law for flow past a porous structure. The bvp is solved using the eigenfunction expansion method in conjunction with matching conditions, giving rise to an over-determined system of linear equations. Least square method is applied and the numerical values of the force on the tunnel for different parameters are obtained and plotted through different graphs. The study reveals that more energy loss and less force on the tunnel are obtained if the porous effect parameter of the plate is increased up to a moderate value. Compared to the case without porous plate and trench-type bottom topography, there are significant changes in forces due to this porous breakwater and trench-type bottom topography, which is helpful in understanding the role of porous breakwaters and trenches in applications to Ocean and Coastal Engineering.

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NONSYMMETRIC INTERIOR PENALTY GALERKIN METHOD FOR BLACK-SCHOLES PDE MODELLING EUROPEAN OPTIONS

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Options are the most important financial derivatives in the financial market and pricing the option is an important task. We discuss an effective numerical approach to the Black-Scholes PDE governing European options in this article.

We are considering the following Equation of Black-Scholes for European Options

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 \mathcal{S}^2 \frac{\partial^2 V}{\partial \mathcal{S}^2} + r\mathcal{S}\frac{\partial V}{\partial \mathcal{S}} - rV = 0, \qquad (\mathcal{S}, t) \in (0, \infty) \times (0, T]\,, \tag{1}$$

with the final conditions

$$V(\mathcal{S}, T) = \begin{cases} (\mathcal{S} - K, 0)^+, & \text{(call option)}\ , \\ (K - \mathcal{S}, 0)^+, & \text{(put option)}, \end{cases} \tag{2}$$

and the boundary conditions

$$V(0, t) = \begin{cases} 0, \\ Ke^{-r(T-t)} - \mathcal{S}, \end{cases} \quad \text{and} \quad V(\mathcal{S}, t) = \begin{cases} \mathcal{S} - Ke^{-r(T-t)}, & \text{(call)}, \\ 0, & \text{(put)}, \end{cases} \tag{3}$$

where, $(\cdot, \cdot)^+$ represents $\max(\cdot, \cdot)$ and the asset price ($\mathcal{S}$), t and r represent the space variable, time variable, and risk-free interest rate, respectively.

The equation (1) is discretize using the implicit non-symmetric interior penalty Galerkin method, and optimal order estimates are acquired for both the discrete energy-norm and L^2- norm. To test the effectiveness of the strategy, additional numerical experiments are carried out for pricing European call option. The stability and convergence results are also derived which agree with the numerical results.

SYMMETRIC FRACTIONAL ORDER REDUCTION METHOD WITH $L1$ SCHEME ON GRADED MESH FOR TIME FRACTIONAL NONLOCAL DIFFUSION-WAVE EQUATION OF KIRCHHOFF TYPE

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In this talk, we present a linearized fully-discrete scheme for solving a time fractional nonlocal diffusion-wave equation of Kirchhoff type [1]. The scheme is established by using the finite element method in space and the $L1$ scheme in time. We discuss the α-robust *a priori* bound and *a priori* error estimate for the fully-discrete solution in $L^\infty(H_0^1(\Omega))$ norm, where $\alpha \in (1,2)$ is the order of time fractional derivative. Finally, we show some numerical experiments to confirm the theoretical results.

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SYNERGISTIC EFFECTS OF DARCY AND VISCOUS DISSIPATION IN NANOFLUID FLOW OVER A VERTICALLY PERMEABLE PLATE WITH MAGNETIC FIELD INDUCTION

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Abstract

This study delves into the theoretical exploration of the impact of nanofluid (CuO/H_2O) flow over a vertical permeable plate without time dependence. The analysis encompasses various factors, including Darcy dissipation, porosity effects, viscous dissipation, local radiant absorption, and heat source/sink phenomena in characterizing heat transfer. Additionally, the magnetic force is introduced to control the expansion of magnetic intensity and the momentum of the flow. The research focuses on the flow of nanofluid with a constant free-stream velocity along an infinitely tall vertical sheet. By employing similarity variables, the original nonlinear equations are transformed into a system of non-dimensional equations. The Runge-Kutta-Fehlberg 45 (RKF-45) numerical method is applied to solve these equations. Furthermore, MATLAB software, specifically the bvpc4c program, is used to investigate the impact of dimensionless parameters on the flow and heat transport. The study examines and discusses the local rate of heat transfer and drag forces. Ultimately, this research underscores the significance of Darcy dissipation, radiant constraint, magnetic Prandtl number, and magnetic intensity in nanofluid flow, with implications for a wide range of engineering applications and cooling processes, as well as in physics.

Keywords: Darcy dissipation; permeability; RKF-45 method; viscous dissipation; nanofluid; thermal radiation

NIPG METHOD FOR NONLINEAR TIME-FRACTIONAL INTEGRO-PARTIAL DIFFERENTIAL EQUATIONS

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We discusses an numerical method for solving nonlinear time-fractional integro-partial differential initial-boundary-value problems. To tackle the non-linearity of the problem, we first use the Newton linearization process. Then we apply the non-symmetric interior penalty Galerkin method for the spatial variable, and obtain a semi-discrete problem in the time variable. Finally, to obtain the fully-discrete scheme, we use both L1-scheme and L2-scheme for time-fractional derivative, and trapezoidal rule for integral term in the semi-discrete problem. We derive L^2-norm stability and error estimates. The validity of the error analysis is supported by numerical experiments.

DEVELOPMENT OF A MODEL FOR DRUG DELIVERY INTO THE DISEASED TISSUES BY REVERSIBLE ELECTROPORATION

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Drug delivery through electroporation could be highly beneficial for the treatment of different types of diseased tissues within the human body. In this work, a mathematical model of reversible tissue electroporation is presented for injecting drug into the diseased cells. The model emphasizes the tissue boundary where the drug is injected as a point source. Drug loss from the tissue boundaries through extracellular space is studied. Multiple pulses are applied to deliver a sufficient amount of drug into the targeted cells. The set of differential equations that model the physical circumstances are solved numerically. This model obtains a mass transfer coefficient in terms of pore fraction coefficient and drug permeability. It controls the drug transport from extracellular to intracellular space. The drug penetration throughout the tissue is captured for the application of different pulses. The boundary effects on drug concentration are highlighted in this study. The advocated model is able to perform homogeneous drug transport into the cells so that the affected tissue is treated completely. This model can be applied to optimize clinical experiments by avoiding the lengthy and costly in vivo and in vitro experiments.

AN EFFICIENT HIGH ORDER QUADRATURE FOR WEAKLY SINGULAR OPERATORS AND ITS APPLICATIONS IN WAVE SCATTERING PROBLEMS

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Weakly singular integral equations appear in many areas of science and engineering, such as wave scattering problems, fractional Laplacian and fluid flow, to name a few. Due to the singularity present in the kernel, classical quadrature rules failed to converge in high order for such integrals. In this talk, we discuss a spectrally accurate numerical integration scheme for weakly singular integrals. We also present a convergence analysis of the quadrature and illustrate the high-order character with various numerical results. Our integration scheme can be easily used as a Nyström solver for the wave scattering problem. An essential feature of our solver is that, unlike the existing method, it maintains high-order convergence even for discontinuous scattering objects.

A Multiphase Model on Asymmetric Growth of an Avascular Tumour

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Abstract

In this article, a one-dimensional (1D) multiphase mathematical model is developed to capture the asymmetric growth of a tumour. In human body, oxygen is not uniformly distributed throughout the healthy and tumour tissue. To date, it remains to investigate how different oxygen concentrations at tumour boundaries affect tumour shape evolution. This model provides a bridge to capture symmetric as well as asymmetric tumour growth that depends on the profile of oxygen concentration at the left and right boundaries. A numerical method is used based on Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) framework to solve the governing equations of the model. The simulation results reveal that the tumour gets an asymmetrical shape when it grows under different oxygen concentrations at two ends, otherwise becomes symmetric. Investigations reveal that the position of necrotic region varies with time as the tumour grows with unequal oxygen concentrations at two edges. Such behaviour may help clinicians, and the medical community to predict the current status of tumour and suitable therapeutic strategies to control the tumour growth.

Keywords: Asymmetric, avascular tumour, multiphase model, Stokes equation, SIMPLE algorithm, FDM.

AN EFFICIENT COMPUTATIONAL TECHNIQUE FOR SEMILINEAR TIME–FRACTIONAL DIFFUSION EQUATION

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This work presents the semi-analytical and the numerical solution of a semi-linear time-fractional diffusion equation. The time-fractional term includes the combination of tempered fractional derivative and k-Caputo fractional derivative with a parameter $k \geq 1$. The application of the new integral transform, namely the Elzaki transform of the tempered k-Caputo fractional derivative is addressed here and thereafter the semi-analytical solution is obtained by using the Elzaki decomposition method. The model problem is linearized using Newton's quasilinearization method, and then the quasilinearized problem is discretized by a difference scheme namely tempered ${}_kL2\text{-}1_\sigma$ method in the uniform mesh. Stability and convergence result of the proposed scheme are presented in the L_2-norm. In support of the theoretical results, numerical example is incorporated.

INTRACELLULAR DYNAMICS OF HEPATITIS B VIRUS INFECTION: A MATHEMATICAL MODEL AND GLOBAL SENSITIVITY ANALYSIS OF ITS PARAMETERS

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Analysis of cell populations provides collective overview on viral infection. On the other hand, single-cell analysis offers a more detailed understanding, allowing for the exploration of individual nuances in the context of infection. This study introduces a comprehensive model for hepatitis B virus infection at the single-cell level, incorporating all discernible intracellular steps observed in the viral life cycle. To the based of our knowledge, this model represents the most generalized framework so far. This model elucidates the impacts of newly introduced viral components, including cccDNA, HBx proteins, surface proteins, and double-stranded linear DNA-containing capsids. In order to find the most sensitive parameter, a global sensitivity analysis is conducted, using partial rank correlation coefficients based on the Latin hypercube sampling method. According to PRCC values, the most positive and most negative sensitive parameters are identified. Moreover, it is observed that availability of viral surface proteins switches the replication pattern from acute to chronic, whereas there is no considerable contribution of HBx proteins to the progression of HBV infection. Another striking result is that the recycling of capsids appears to act as a positive feedback loop throughout the infection.