GROUND STATE SOLUTION OF A SCHÖRDINGER TYPE EQUATION INVOLVING ANISOTROPIC P-LAPLACIAN OPERATOR VIA POHOZǍEV MANIFOLD

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In this talk we will discuss the existence of a positive ground-state solution of the following Schrödinger equation by using the Pohozǎev manifold:

$$\begin{cases} -\Delta_{H,p}u + V(x)|u|^{p-2}u - \Delta_{H,p}(|u|^{2\alpha})|u|^{2\alpha-2}u = \lambda|u|^{q-1}u \text{ in } \mathbb{R}^n \\ u \in W^{1,p}(\mathbb{R}^n) \end{cases} \tag{1}$$

where $\mathbb{N} \geq 2$, $(\alpha, p) \in D_N = \{(x,y) \in \mathbb{R}^2 : 2xy \geq y+1,\ y \geq 2x,\ y < N\}$ and $\lambda > 0$ is a parameter. The operator $\Delta_{H,p}$ is defined as

$$\Delta_{H,p}u := \operatorname{div}(H(Du)^{p-1}\nabla_\eta H(Du))$$

known as Finsler or anisotropic p-Laplacian, where ∇_η denotes the gradient operator with respect to η variable. And the norm function $H : \mathbb{R}^N \to [0, \infty)$ and the potential V satisfy few technical properties.

DISCRETE TIME-DEPENDENT WAVE EQUATION FOR THE SCHRÖDINGER OPERATOR WITH UNBOUNDED POTENTIAL

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In this article, we investigate the semiclassical version of the wave equation for the discrete Schrödinger operator, $\mathcal{H}_{\hbar,V} := -\hbar^{-2}\mathcal{L}_\hbar + V$ on the lattice $\hbar\mathbb{Z}^n$, where $\mathcal{L}_\hbar$ is the discrete Laplacian, and V is a non-negative multiplication operator. We prove that $\mathcal{H}_{\hbar,V}$ has a purely discrete spectrum when the potential V satisfies the condition $|V(k)| \to \infty$ as $|k| \to \infty$. We also show that the Cauchy problem with regular coefficients is well-posed in the associated Sobolev type spaces and very weakly well-posed for distributional coefficients. Finally, we recover the classical solution as well as the very weak solution in certain Sobolev type spaces as the limit of the semiclassical parameter $\hbar \to 0$.

POSITIVE SOLUTIONS FOR FRACTIONAL P- LAPLACE PROBLEM WITH INDEFINITE SIGN

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In this talk, we consider a fractional p-Laplace problem of the form

$$(P^\mu)\begin{cases} (-\Delta)_p^s u &= \mu f(u) \quad \text{in } \Omega, \\ \qquad u &= 0 \qquad\quad \text{on } \Omega^c, \end{cases}$$

where Ω is a bounded smooth domain, and μ a positive parameter. The nonlinearity $f(t)$ is a sign changing continuous function with $f(0) < 0$, and admits a subcritical growth at infinity. We study of the behavior of a specific barrier function when acted upon by the fractional p-Laplace operator. This investigation leads us to establish a crucial L^∞ estimate, which in turn is employed to demonstrate the existence of a solution for (P^μ) for small μ. Since $f(0) < 0$, the significant challenge is to establish the positivity of the solution which we address by applying fine boundary regularity and Hopf Lemma. Additionally, this talk explores the existence of a ground state positive solution for a multiparameter semipositone problem with critical growth using variational arguments.

ON A CLASS OF INFINITE SEMIPOSITONE PROBLEMS FOR (P, Q) LAPLACE OPERATOR

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The presentation centers on investigating the existence of a positive solution to the following non-linear elliptic boundary value problem

$$-\Delta_p u - \Delta_q u = \lambda \frac{f(u)}{u^\beta} \text{ in } \Omega$$
$$u = 0 \text{ on } \partial\Omega$$

in an arbitrary bounded domain in $\mathbb{R}^N$ having a smooth boundary. The non-linearity here is driven by a singular, monotonically increasing continuous function in $(0, \infty)$ which is eventually positive. The distinctive nature of the non-linearity, known as the "infinite semipositone", coupled with the non-homogeneity of the operator makes the mathematical analysis challenging. Moreover, the absence of symmetry in the domain compels us to construct a non-trivial sub-solution. However, by applying a fixed point theorem we establish the existence of a positive solution to this problem. Further, in a specific scenario, we prove this solution is, in fact, a maximal positive solution. Lastly, we delve into the precise asymptotic behavior of solutions for another problem when the parameter takes on large values. This result is of independent interest and can be employed to showcase a uniqueness result.

NEHARI MANIFOLD METHOD FOR SINGULAR DOUBLE PHASE PROBLEM WITH OPTIMAL CONTROL ON PARAMETER

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In this study we consider the double phase problem $-\mathrm{div}(|\nabla u|^{p-2}\nabla u + a(x)|\nabla u|^{q-2}\nabla u)$ involving a positive parameter λ with singular and subcritical nonlinearity containing sign changing weight, in Ω, where $\Omega \subset \mathbb{R}^N$ is an open bounded domain with smooth boundary, $N \geq 2$, $1 < p < q$ and the modulating function a is non-negative, continuous and has compact support Ω. Using fibering map and Nehari manifold method, we show the existence of at least two positive solutions when $\lambda \in (0, \lambda^* + \epsilon)$ for some $\epsilon > 0$, where λ^* is an extremal parameter, characterized via nonlinear Rayleigh quotient. A finer analysis has been done once λ crosses the extremal value λ^*.

EXISTENCE AND MULTIPLICITY OF POSITIVE SOLUTIONS FOR A CLASS OF ELLIPTIC EQUATIONS INVOLVING SUBCRITICAL AND CRITICAL NON LINEARITIES IN THE HYPERBOLIC SPACE

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In recent years, many efforts have been made to generalise scalar field-type equations in the domains after their connection with various physical problems. Hyperbolic space is the simplest model motivated by the problem of prescribing curvature in the non-compact setting. In the hyperbolic space, Sandeep-Mancini have shown the existence and uniqueness of finite energy positive solutions of a homogeneous elliptic equation. Inspired by this, we have been investigating whether positive solutions for a perturbed problem can still exist. Precisely, we studied the existence and multiplicity of positive solutions for the following class of scalar field problem

$$-\Delta_{\mathbb{B}^N} u - \lambda u = a(x)|u|^{p-1}\, u + f(x,u) \ \text{ in } \mathbb{B}^N, \quad u \in H^1(\mathbb{B}^N), \tag{1}$$

where $\mathbb{B}^N$ denotes the hyperbolic space, $1 < p \leq 2^* - 1 := \frac{N+2}{N-2}$, if $N \geqslant 3; 1 < p < +\infty$, if $N = 2$, $\lambda < \frac{(N-1)^2}{4}$, and $0 < a \in L^\infty(\mathbb{B}^N)$, and $f(x,u)$ is a class of perturbed functions. Precisely, we considered the cases when p is in the subcritical range and $f \equiv 0$, or f is a non-homogeneous term, or f is of the type $\varepsilon |u|^{2^*-2}u$, for ε sufficiently small. Furthermore, we have also explored the purely critical case, i.e., $p = 2^* - 1$ and $f \equiv 0$. This variational problem lacks compactness even in the subcritical case because of the *hyperbolic translation*. Additionally, for the critical exponent, the loss of compactness can happen along two different profiles, one along the hyperbolic translations and the other along concentration of Aubin-Talenti bubble (locally) making it futile to solve using the standard minimization method. Moreover, (1) is not symmetric in the hyperbolic space due to the $a(x)$ term, so the ODE techniques failed. Thus to study (1), we start by characterizing the Palais-Smale sequences associated with (1), then explore a few energy estimates involving hyperbolic and Aubin-Talenti bubbles. The evaluation of these estimates needed to be tackled very delicately since the hyperbolic volume grows exponentially with the radius, i.e., $\mathrm{d}V_{\mathbb{B}^N} \asymp e^{(N-1)r}$, $r \to \infty$ unlike the polynomial growth r^{N-1} of volume in $\mathbb{R}^{\mathbb{N}}$. Finally, we prove the existence and multiplicity of positive solutions by using various techniques to overcome the compactness (below some 'critical' level) like the min-max procedure in the spirit of Bahri-Li, appropriately defining a barycentric map and using *topological degree* arguments, LS category theory, mountain pass theorem etc. Moreover, we also studied asymptotic estimates of solutions to (1) for $\lambda \leq 0$, $a(x) \equiv 1$, p subcritical and f satisfies some decay estimates by following the approach of constructing suitable barriers as sub and super solutions. When $f \equiv 0$, we recovered the optimal estimates obtained by Sandeep-Mancini for radial solutions.

MATHEMATICAL ANALYSIS OF COUPLED-MODE THEORY

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Many phenomena in science/engineering involve the coupling of two or more systems, in which the dynamics of one system affect the dynamics of the other systems, e.g., resonance circuits, coupled pendulums, coupled mass springs, etc. Mathematical models of such systems have some patterns and symmetries that provide insights into the dynamics of the coupled systems. In this talk, we discuss mathematical aspects of the coupled phenomena, which are modeled by a system of first-order differential equations.

One way to deal with such a system is to solve the governing differential equations with suitable initial and boundary conditions. This may be complicated sometimes. Hence, a general procedure is sought for dealing with such coupled systems, which is effective and easy to handle.

Coupled-mode theory is a general phenomenological approach that works for such systems. Haus has given phenomenological equations that describe the coupling between two systems:

$$\begin{aligned} \frac{da_1}{dt} &= j\omega_1 a_1 + \kappa_{12} a_2 \\ \frac{da_2}{dt} &= \kappa_{21} a_1 + j\omega_2 a_2 \end{aligned} \tag{1}$$

where a_1, a_2 are the eigenfunctions for the eigenvalues ω_1, ω_2 of the uncoupled system and κ_{12}, κ_{21} are the unknown parameters representing the effect of interaction between a_1, a_2. Researchers have found that this model is applicable for weak coupling settings; there is a need for an improved model for non-weak coupling settings.

In this talk, we present a mathematical framework to derive the exact coupled-mode theory equations. We will show analytically that the coupled-mode theory equations represent a first-order approximated model that works for weakly coupled systems. A reduced model is obtained from the exact model by retaining appropriate correction terms in the natural frequencies ω_1 and ω_2. The reduced model also facilitates obtaining exact expressions for the parameters κ_{12}, κ_{21}. The numerical analysis suggests the reduced model gives more accurate results than the previous model (1).

FINITE MEIJER G-TRANSFORM FOR SOLVING STEADY-STATE TEMPERATURE PROBLEMS

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The operational properties of the finite Meijer G-transform, with solutions to boundary value problems using partial differential equations are studied. Few applications in Mathematical Physics are demonstrated on potential and steady-state temperature.

Study of unsteady solute dispersion in pulsatile viscoelastic fluid flow

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Abstract

The axial solute dispersion in a pulsatile viscoelastic fluid flow through a straight tube is studied in the presence of absorption at the tube wall. This analysis is suitable for understanding the transportation of the drug in the blood flow to get its most therapeutic effects in the treatment of the patients. The objective of the present study is to understand the convection, dispersion and concentration of the solute. The blood vessel is assumed to be a straight, circular, cylindrical tube. The pulsatile nature of the blood is considered for unsteady flow owing to the rhythmic pumping by the heart. The blood rheology is highly influenced by the red cells, and it can be aggregated, deformed and aligned in a single file with different shear rates. At low shear rates, the viscoelastic behaviour is experienced during the blood flow. This property of the blood is characterized by the Oldroyd-B fluid model. The velocity profile for Oldroyd-B fluid is obtained after solving the momentum equation. Both analytical and numerical investigations are made to solve the convective diffusion equation and analyze the solute dispersion phenomenon at small and large times after the solute injection in the blood flow. The exchange, convection, dispersion and higher-order transport coefficients, which describe the transportation process in the system, are determined. More importantly, the mean concentration of the solute is analyzed. The skewness and kurtosis are also examined to discuss the non-Gaussian distribution of the solute. The effect of the viscoelastic characteristics of the fluid on the solute dispersion process is noted in this work.

FRACTIONAL MODEL FOR BLOOD FLOW IN A STENOSED ARTERY UNDER MHD EFFECT THROUGH POROUS MEDIUM

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In this article, we present a fractional version of the model for blood flow in a stenosed artery under the MHD (magnetohydrodynamic) effect through a porous medium. Here, we consider a cosine-shaped stenosis inside the artery and under the light of fractional calculus we present not only an analytical solution to the model but also, we carry out a rigorous graphical analysis followed by verification, validation and drawing of some interesting attributes to the model.

L^∞ ESTIMATE FOR DEGENERATE TRANSVERSE COMPLEX MONGE-AMPÈRE EQUATION AND ITS APPLICATION

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Let (X, ω) be a Kähler manifold and $F : X \to \mathbf{R}$ be a smooth function such that $\int_X e^F \omega^n = \int_X \omega^n$, then the complex Monge-Ampère equation is defined as

$$(\omega + \sqrt{-1}\partial\bar{\partial}\varphi)^n = e^F \omega^n, \ \omega + \sqrt{-1}\partial\bar{\partial}\varphi > 0.$$

The L^∞ estimate was the hardest part in solving the complex Monge-Ampère equation and it was achieved by Yau by assuming that the right hand-side is in L^p for $p > n$, where n is the complex dimension of the manifold. And it was improved by Kolodziej assuming that the right hand-side is in L^p, for $p > 1$ and his proof relied completely on techniques from pluripotential theory.

In this talk we provide the sharp L^∞ estimate for the degenerate transverse complex Monge-Ampère equation on *transverse nef* class with right hand side is in $L^1(\log L)^p$, for $p > n$, by extending the PDE techniques developed by Guo-Phong-Tong-Wang [2] to transverse Kähler manifold. And as an application we prove that the conical Calabi-Yau potential on the $\mathcal{Q}$-Gorenstein normal affine variety is smooth on the regular locus, this provides the complete PDE proof of [1, 3].

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BOUNDARY BLOW UP SOLUTIONS INVOLVING FRACTIONAL P-LAPLACIAN OPERATOR

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In this article we study the existence of boundary blow up solutions for equation involving fractional p-Laplacian operator. We consider the boundary value problem

$$(P)\begin{cases} (-\Delta)_p^s u + u^q &= f \qquad \text{in } \Omega, \\ u &= g \qquad \text{on } \Omega^c, \\ \lim\limits_{x\in\Omega, x\to\partial\Omega} u &= \infty, \end{cases}$$

where Ω is bounded domain of class C^2 in $\mathbb{R}^N$, $\Omega^c := \mathbb{R}^N \setminus \Omega$, $p \in (1,\infty)$, $q > p-1+sp$, $s \in (0,1)$ such that $N > sp$ and the functions $f : \Omega \to \mathbb{R}$ and $g : \overline{\Omega}^c \to \mathbb{R}$ are continuous. We obtain the existence of a solution u when the boundary value g blows up at the boundary, and we get the explosion rate for u under an assumption on the rate of explosion of g.