

- (1) **Speaker:** Aditya Karnataki, Chennai Mathematical Institute
Title: Families of (φ, τ) -modules and Galois representations
Abstract: Let K be a finite extension of \mathbb{Q}_p . The theory of (φ, Γ) -modules constructed by Fontaine provides a good category to study p -adic representations of the absolute Galois group $Gal(\bar{K}/K)$. This theory arises from a “devissage” of the extension \bar{K}/K through an intermediate extension K_∞/K which is the cyclotomic extension of K . The notion of (φ, τ) -modules generalizes Fontaine’s constructions by using Kummer extensions other than the cyclotomic one. It encapsulates the important notion of Breuil-Kisin modules among others. It is thus desirable to establish properties of (φ, τ) -modules parallel to the cyclotomic case. In this talk, we explain the construction of a functor that associates to a family of p -adic Galois representations a family of (φ, τ) -modules. The analogous functor in the (φ, Γ) -modules case was constructed by Berger and Colmez. Time permitting, we will indicate some future directions of research in this area. This is joint work with Léo Poyeton.
- (2) **Speaker:** Guhan Venkat, Ashoka University
Title: Rationality of Stark-Heegner points for elliptic curves over imaginary quadratic fields.
Abstract: In this seminar, I will report about a recent result that shows that Stark-Heegner points attached to elliptic curves over imaginary quadratic fields are global in the base-change scenario.
- (3) **Speaker:** Arnab Saha, IIT Gandhinagar
Title: Delta geometry and the characteristic polynomial of the Frobenius
Abstract: By a previous construction by Borger and the speaker, one attaches a canonical filtered isocrystal $\mathbf{H}_\delta(G)$ associated to the arithmetic jet spaces of a smooth commutative group scheme G . In the case when A is an elliptic curve, $\mathbf{H}_\delta(A)$ is isomorphic to the first crystalline cohomology $\mathbf{H}_{\text{cris}}^1(A)$ when A does not have a canonical lift. The above comparison theorem allows us to give a character theoretic interpretation of the crystalline cohomology.
In this talk, using the characteristic polynomial of the Frobenius, we will show that whenever A is an abelian scheme, the module of primitive characters of A is isomorphic to $H^0(A, \Omega_A)$. We will also comment on the relation of $\mathbf{H}_\delta(A)$ with the first crystalline cohomology $\mathbf{H}_{\text{cris}}^1(A)$. This is joint work with L. Gurney and S. Pandit.
- (4) **Speaker:** C. S. Rajan, Ashoka University
Title: On non-commensurable isospectral locally symmetric spaces.
Abstract: We give examples of non-commensurable but isospectral locally symmetric spaces, thereby completing the work of Lubotzky, Samuels and Vishne. The main step is to show that adelic conjugation of lattices in $SL(1, D)$ by the adjoint group preserves the spectrum, where D is a division algebra over a number field F (under some additional hypothesis), extending the work done by one of us, when D is quaternion. This is joint work with Sandeep Varma.
- (5) **Speaker:** Anirban Mukhopadhyay, IIMSc Chennai
Title: Turan-Kubilius inequality
Abstract: Starting from the Kubilius model for additive functions, we shall discuss the Turan-Kubilius inequality for integers and for polynomials over finite fields.

Some variants of the inequality and its applications are part of a joint work with Pranendu Darbar.

- (6) **Speaker:** Sneha Chaubey, IIIT Delhi

Title: Density of visible lattice points along polynomials

Abstract: Recently, the notion of visibility from the origin has been generalized by viewing lattice points through curved lines of sights. In this talk, we discuss the notion of visible lattice points for any polynomial family of curves passing through the origin. We show that except for the family of curves $\{y = qx^k | q \in \mathbb{Q}^+, k \in \mathbb{N}\}$, the density of visible lattice points for any other polynomial family of curves passing through the origin is one.

- (7) **Speaker:** Kummari Malleshm, IIT Bombay

Title: General Rankin-Selberg problem.

Abstract: In analytic number theory, it is a very fundamental question to understand the summatory function

$$S(X) = \sum_{1 \leq n \leq X} a(n)$$

of an arithmetical function $(a(n))_{n=1}^{\infty}$. In this talk, we discuss about bounds for $S(X)$ when $a(n)$'s are given by Fourier coefficients of Rankin-Selberg L -function of holomorphic cusp forms f and g . The content of the talk is based on a joint work with Aritra Ghosh, Ritabrata Munshi and Saurabh Kumar Singh.

- (8) **Speaker:** Sudhir Pujahari, NISER Bhubaneswar

Title: An all purpose Erdős-Kac Theorem.

Abstract: In 1917, Hardy and Ramanujan showed that any natural number has a $\log \log n$ number of distinct prime factors. The above work led Erdős and Kac to their famous result known as "Erdős-Kac theorem" where they discovered the Gaussian law for distinct prime divisors of natural numbers. These results ignited the development of probabilistic number theory. In this talk, we will see an all purpose Erdős-Kac theorem and apply it in different settings. One of the interesting applications we will focus on is an Erdős-Kac theorem for sums of Fourier coefficients of Hecke eigenforms. This is a joint work with Ram Murty and Kumar Murty.

- (9) **Speaker:** Saranya G. Nair, BITS Pilani, Goa Campus

Title: Newton Polygons and the constant associated with the Prouhet-Tarry-Escott Problem

Abstract: Given natural numbers n and k , with $n > k$, the Prouhet-Tarry-Escott (PTE) problem asks for distinct sets of integers, say $X = [x_1, x_2, \dots, x_n]$ and $Y = [y_1, y_2, \dots, y_n]$, such that

$$\sum_{i=1}^n x_i^j = \sum_{i=1}^n y_i^j \text{ for } j = 1, 2, \dots, k.$$

We discuss new information on the lower bounds of 2-adic valuation of certain constants \overline{C}_n associated with the PTE problem for the cases $n = 10$ and 12 using the classical theory of Newton Polygons. This is a joint work with Dr. Ranjan Bera.

- (10) **Speaker:** Shamik Ghosh, Jadavpur University, Kolkata.

Title: The Goldbach Graph

Abstract: In a study of oriented bipartite graph we introduced the concept of odd-even graphs and digraphs in 2021. It was shown that every bipartite graph and every acyclic oriented bipartite graph can be represented by odd-even graph and digraph respectively. The Goldbach graph is an odd-even graph whose vertex set is the set of all non-negative even integers and two such integers a and b are adjacent in the graph if and only if both $\frac{a+b}{2}$ and $\frac{|a-b|}{2}$ are odd prime numbers. A Goldbach finite graph is an induced subgraph of the Goldbach graph with finite number of vertices. In this talk we describe many interesting properties of the Goldbach graph and its finite subgraphs. Most interestingly, the famous Goldbach conjecture is equivalent to the connectedness of Goldbach finite graphs. Several other similar number theoretic conjectures are related to various parameters of Goldbach graphs. Finally, we observe Hamiltonian properties of some odd-even graph which is closely related to Goldbach finite graphs for small number of vertices. This gives a sequence of even integers in which for any two consecutive numbers in the sequence, if one is the sum of two odd primes (or, 1), then the other is the difference of them.

- (11) **Speaker:** Neelam Saikia, IIT Bhubaneswar

Title: Distributions of hypergeometric functions

Abstract: In the 1980's, Greene introduced ${}_nF_{n-1}$ hypergeometric functions over finite fields using normalized *Jacobi sums*. The framework of his theory provides that these functions possess many properties that are analogous to those of the classical hypergeometric series studied by Gauss, Kummer and others. In this talk, we discuss the value distributions of certain "simplest" families of these hypergeometric functions. For the ${}_2F_1$ functions, the limiting distribution is semicircular, whereas the distribution for the ${}_3F_2$ functions is *Batman* distribution. This is a joint work with Ken Ono and Hasan Saad.

- (12) **Speaker:** Gautam Kalita, Indian Institute of Information Technology Guwahati

Title: Supercharacter theory on \mathbb{F}_p^2 and related exponential sums

Abstract: Building upon the work of André [1] and Yan [3] on the representation theory of the group of unipotent upper triangular matrices $UT_n(\mathbb{F}_q)$ over the finite field \mathbb{F}_q , Diaconis and Isaacs [2] introduced the concept of supercharacter theory as generalization of basic characters.

Definition 0.1. [2] Let G be a finite group, let \mathcal{X} be a partition of the set $\text{Irr}(G)$ of irreducible characters of G , and let \mathcal{Y} be a partition of G . We call the ordered pair $(\mathcal{X}, \mathcal{Y})$ a supercharacter theory if

- (i) \mathcal{Y} contains $\{1\}$, where 1 denotes the identity element of G ,
- (ii) $|\mathcal{X}| = |\mathcal{Y}|$,
- (iii) for each $X \in \mathcal{X}$, the character

$$\sigma_X = \sum_{\chi \in X} \chi(1)\chi$$

is constant on each $Y \in \mathcal{Y}$.

The characters σ_X are called supercharacters and the elements $Y \in \mathcal{Y}$ are called superclasses.

Computation of a supercharacter theory for a finite group is a problem of interest for mathematicians, and supercharacter theories for several finite groups have already been deduced. Supercharacter theories of abelian groups are much simpler compared to those of other groups. Supercharacter theories on the abelian group $(\mathbb{Z}/n\mathbb{Z})^d$ induced by the action of certain subgroups of the matrix group $GL_d(\mathbb{Z}/n\mathbb{Z})$ have been studied extensively, and showed that many exponential sums of number theoretical importance can be expressed as supercharacter values of these supercharacter theories.

In this talk, we consider certain supercharacter theories on \mathbb{F}_p^2 for which some exponential sums appear as supercharacter values. Then, we explore connections of power moments of these exponential sums with elliptic curves. We further discuss some alternative proofs of many well-known results on exponential sums together with some new results concerning them.

REFERENCES

- [1] C. A. M. André, *The basic character table of the unitriangular group*, J. Algebra **241** (2001), no. 1, 437–471.
- [2] P. Diaconis and I. M. Isaacs, *Supercharacters and superclasses for algebra groups*, Trans. Amer. Math. Soc. **360** (2008), no. 5, 2359–2392.
- [3] N. Yan, *Representation Theory of the finite unipotent linear groups*, Unpublished manuscript, (2001).

(13) **Speaker:** Atul Dixit, IIT Gandhinagar

Title: Applications of the Lipschitz summation formula and a generalization of Raabe’s cosine transform

Abstract: General summation formulas have been proved to be very useful in analysis, number theory and other branches of mathematics. The Lipschitz summation formula is one of them. In this talk, we employ this summation formula to obtain a new transformation which generalizes a transformation of Ramanujan. Ramanujan’s result, in turn, is a generalization of the modular transformation of Eisenstein series $E_k(z)$, $k \geq 2$, on $SL_2(\mathbb{Z})$, where $z \rightarrow -1/z, z \in \mathbb{H}$. The proof of our generalization involves a delicate analysis containing Cauchy Principal Value integrals. A simpler proof of our recent result with Kesarwani, which gives a non-modular transformation for $\sum_{n=1}^{\infty} \sigma_{2m}(n)e^{-ny}$, is also derived using the Lipschitz summation formula. In the pursuit of obtaining this transformation, we naturally encounter a new generalization of Raabe’s cosine transform whose several properties are also demonstrated. As an application of our results, we derive a generalization of Wright’s asymptotic estimate for the generating function of the number of plane partitions of a positive integer n . This is joint work with Rahul Kumar.

(14) **Speaker:** Divyum Sharma, BITS Pilani, Pilani Campus

Title: Number of solutions of Thue inequalities

Abstract: Let $F(x, y) \in \mathbb{Z}[x, y]$ be an irreducible form of degree $r \geq 3$ and having $s + 1$ non-zero coefficients. Let h be a positive integer and consider the inequality

$$|F(x, y)| \leq h.$$

Following the seminal work of Thue in 1909, a significant amount of work has been done to provide upper bounds for the number of integer solutions of the above inequality which are independent of the coefficients of F . In this talk, we present some recent bounds, with a special focus on *diagonalizable* forms. This is based on joint

works with N. Saradha.

(15) **Speaker:** Bipul Kumar Sarmah, Tezpur University

Title: Dissection of Euler's product

Abstract: For any complex number a , we define

$$(a; q)_k := \prod_{n=1}^k (1 - aq^{n-1})$$

and

$$(a; q)_\infty := \lim_{k \rightarrow \infty} (a; q)_k = \prod_{n=1}^{\infty} (1 - aq^{n-1}), \quad |q| < 1.$$

We set $E(q) := (q; q)_\infty = \prod_{n=1}^{\infty} (1 - q^n)$. $E(q)$ is known as Euler's product.

A t -dissection of any power series $A(q)$ is given by $A(q) = \sum_{k=0}^{t-1} q^k A_k(q^t)$, where

$A_k(q^t)$ are power series in q^t . Dissection of various combination of Euler's product is an important tool in additive number theory. M. D. Hirschhorn in his book 'The Power of q ' gave the 2- and 4- dissections of $E(q)$ and gave a engrossing conjecture on 2^n - dissection of $E(q)$. We present an elementary proof of that conjecture by using properties of Ramanujan's theta functions.

(16) **Speaker:** U. K. Anandavardhanan ,IIT Bombay

Title: Distinguished representations for Galois pairs

Abstract: In this talk we'll give a broad introduction to the topic of distinguished representations which are the central objects of study in the relative Langlands program. We'll motivate the question by first looking at finite groups and then introduce the corresponding notions for p-adic and adelic groups. We'll survey some of the results for $GL(n)$ and then state a few recent results for $SL(n)$, where the study is more subtle due to the presence of L-packets. The new results in the talk are joint with Nadir Matringe and Dipendra Prasad.