JOIN OF AFFINE SEMIGROUPS

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We introduce the notion of an affine semigroup generated by join of two affine semigroups and show that it preserves some nice properties, including Cohen-Macaulayness, when the constituent semigroups have those properties. This is a joint work with Joydip Saha and Pranjal Srivastava.

ON COMPONENTWISE LINEARITY OF EDGE IDEALS OF WEIGHTED ORIENTED GRAPHS

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The study of ideals having linear resolution has paid lots of attention among researchers in Commutative Algebra. These ideals behave nicely and computationally simple than others. One of the remarkable results due to Eagon-Reiner says that the Stanley-Reisner ideal of a simplicial complex is Cohen-Macaulay if and only if its Alexander dual has a linear resolution. Componentwise linear ideals, which is a large class of ideals including the class of ideals having linear resolution, has become a popular in the theory of Commutative Algebra since they enjoy various nice properties and algebraic interpretation. In 1990, Fröberg established a significant result stating that the edge ideal of a simple graph G, denoted as $I(G)$, has a linear resolution (or componentwise linear) if and only if the complement graph of G is chordal.

In this talk, we discuss about the componentwise linearity of edge ideals of weighted oriented graphs (WOGs). It is an open question that which weighted oriented graphs are componentwise linear ? Can we characterize them combinatorially ? We can not simply generalize or extend characterizations of componentwise linear simple graphs to WOGs because edge ideals of WOGs are not square-free, not equigenerated and depends on the orientation and weights. We showed various combinatorial characterizations for componentwise linearity of WOGs.

A CRITERION TO DETERMINE RESIDUAL COORDINATES OF $\mathbb{A}^2$-FIBRATIONS

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Given an element of a polynomial algebra, checking whether it is a coordinate is a difficult problem, especially when the base ring is not a field. The problem is not easy even for the two-variable polynomial algebras. To handle such problems, in 1988, Bhatwadekar introduced the concept of residual variables, and later, in 1993, Bhatwadekar-Dutta systematically developed the theory of residual coordinates of polynomial algebras. Since the problem of affine fibration asks whether a given affine fibration is a polynomial algebra, to tackle the related problems, the concept of residual coordinates of affine fibrations was introduced in 2013 by Kahoui-Oual, and a detailed theory was developed in 2015 by Das-Dutta which subsequently helped to tackle many problems in affine fibrations.

However, determining whether an element is a residual coordinate requires a significant effort. In 1997, Bhatwadekar-Dutta established a useful criterion to determine residual variables, which states that an element of a polynomial algebra in two variables over a Noetherian domain containing $\mathbb{Q}$ is a residual coordinate if and only if the partial derivatives of that element form the unit ideal. In this talk, we discuss a criterion akin to Bhatwadekar-Dutta's criterion that helps determine whether an element of an $\mathbb{A}^2$ fibration is a residual coordinate.

REES ALGEBRA OF MAXIMAL ORDER PFAFFIANS AND ITS DIAGONAL SUBALGEBRAS

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Given a skew-symmetric matrix X, the Pfaffian of X is defined as the square root of the determinant of X. In this article, we give the explicit defining equations of the Rees algebra of a Pfaffian ideal I generated by the maximal order Pfaffians of a generic skew-symmetric matrix. We further prove that all diagonal subalgebras of the corresponding Rees algebra of I are Koszul. We also look at Rees algebras of Pfaffian ideals of linear type associated with certain sparse skew-symmetric matrices. In particular, we consider the tridiagonal matrices and identify the corresponding Pfaffian

ideals to be of Gröbner linear type and as the vertex cover ideals of unmixed bipartite graphs. As an application of our results, we conclude that all their ordinary and symbolic powers have linear quotients.

NOETHERIAN SYMBOLIC REES ALGEBRAS

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In this paper we extend a result of Cowsik on set-theoretic complete intersection and a result Huneke, Morales and Goto and Nishida about Noetherian symbolic Rees algebras of ideals. As applications, we show that the symbolic Rees algebras of the following ideals are Noetherian and the ideals are set-theoretic complete intersections: (a) the edge ideal of a complete graph, (b) the Fermat ideal and (c) the Jacobian ideal of a certain hyperplane arrangement.

BUCHSBAUMNESS OF THE ASSOCIATED GRADED RINGS OF FILTRATIONS

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Suppose $(A, \mathfrak{m})$ is a Buchsbaum local ring of positive dimension and I is an $\mathfrak{m}$-primary ideal. We discuss some conditions for the Buchsbaumness of the associated graded ring $\mathcal{G}$ of an I-good filtration. The I-adic case of our results are already known. In that case, the advantage is that the unique homogeneous maximal ideal of the Rees algebra has degree one generators. Lack of the above property is the major difficulty while dealing with a filtration. However, we find a generating set with certain properties which enables us to generalize many results from literature on the Buchsbaumness of $\mathcal{G}$.

A STUDY OF V-NUMBER FOR SOME MONOMIAL IDEALS

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Let I be a monomial ideal in $S = K[x_1, \ldots, x_t]$ and $v(I)$ denote the v-number of I. In this talk we show that or a monomial ideal I, $v(I^k)$ is bounded above by a linear polynomial for large n and for certain classes of monomial ideals, the upper bound is achieved for all $k \geq 1$. For an $\mathfrak{m}$-primary monomial ideal, we provide an explicit formula for $v(I)$, and give an upper bound of $v(I)$ in terms of the degree of its generators. For $\mathfrak{m}$-primary monomial ideal I we prove that $v(I) \leq \text{reg}(S/I)$ and their difference can be arbitrarily large. We show that for an edge ideal of a graph with n edges $v(I^k) = 2k - 1$ for all $k \geq 1$. Further we prove that for squarefree monomial ideal I with $v(I) = \alpha(I) - 1$, $v(I^k) = k\alpha(I) - 1$ for all $k \geq 1$, where $\alpha(I)$ denote the minimum degree of the generators of I.

SYMBOLIC POWERS OF IDEALS, RESURGENCE AND WALDSCHIMD CONSTANT

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Symbolic powers of ideals have been of recent interest. However, describing the generators of symbolic powers is not easy. In particular one would like to compare the ordinary powers and symbolic powers. To give a better understanding C. Bocci and B. Harbourne defined an asymptotic quantity called resurgence [1]. ince it is hard to compute the exact value of resurgence, in the same paper [1], they defined another invariant was first introduced by Waldschmidt in [4]. They call this invariant the Waldschmidt constant. In this talk we well discuss these invariants for certain curves. This is joint work with [2] and S. Masuti [3].

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BOCKSTEIN COHOMOLOGY OF MCM MODULES OVER GORENSTEIN ISOLATED SINGULARITIES

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Let $(A, \mathfrak{m})$ be an excellent equi-charateristic Gorenstein isolated singularity of dimension $d \geq 2$. Assume the residue field of A is perfect. Let I be any $\mathfrak{m}$-primary ideal. Let $G_I(A) = \bigoplus_{n\geq 0} I^n/I^{n+1}$ be the associsted graded ring of A with respect to I. Let M be a finitely generated A-module. Let $G_I(M) = \bigoplus_{n\geq 0} I^nM/I^{n+1}M$ be the associsted graded ring of M with respect to I (considered as a $G_I(A)$-module). Let $BH^i(G_I(M))$ be the i^{th}-Bockstein cohomology of $G_I(M)$ with respect to $G_I(A)_+$-torsion functor. We show there exists $a \geq 1$ depending only on A such that if I is any $\mathfrak{m}$-primary ideal with $I \subseteq \mathfrak{m}^a$ and $G_I(A)$ generalized Cohen-Macaulay then the Bockstein cohomology $BH^i(G_I(M))$ has finite length for $i = 0, \ldots, d-1$ for any maximal Cohen-Macaulay A-module M.

ON THE RESURGENCE AND ASYMPTOTIC RESURGENCE OF HOMOGENEOUS IDEALS

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Let $\mathbf{k}$ be a field and $R = \mathbf{k}[x_1, \ldots, x_n]$. We obtain an improved upper bound for asymptotic resurgence of squarefree monomial ideals in R. We study the effect on the resurgence when sum, product and intersection of ideals are taken. We obtain sharp upper and lower bounds for the resurgence and asymptotic resurgence of cover ideals of finite simple graphs in terms of associated combinatorial invariants. We also explicitly compute the resurgence and asymptotic resurgence of cover ideals of several classes of graphs. We characterize a graph being bipartite in terms of the resurgence and asymptotic resurgence of edge and cover ideals. We also compute explicitly the resurgence and asymptotic resurgence of edge ideals of some classes of graphs. This is joint work with A.V. Jayanthan and Arvind Kumar.

SOLVABILITY OF THE SK_1-ANALOG OF THE ORTHOGONAL GROUPS

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In this talk, I shall discuss the dilation principle and associated analog of Quillen's local-global principle for the relative Dickson-Siegel-Eichler-Roy (DSER) elementary orthogonal group using the absolute dilation principle. Applying the relative local-global principle, we prove the solvability of the SK_1-analog for orthogonal groups and study the homotopy and commutativity principle for odd elementary orthogonal groups.

A DENSITY FUNCTION OF THE EPSILON MULTIPLICITY

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Suppose that I is an ideal in a Noetherian local ring $(R, \mathfrak{m})$ of Krull dimension d. Ulrich and Validashti defined the ε-multiplicity of I to be

$$\varepsilon(I) := \limsup_{n\to\infty} \frac{\lambda_R\left(H^0_{\mathfrak{m}}(R/I^n)\right)}{n^d/d!}.$$

The ε-multiplicity can be seen as a generalization of the classical Hilbert-Samuel multiplicity. Cutkosky showed that the 'lim sup' in the definition of ε-multiplicity can be replaced by a limit if the local ring $(R, \mathfrak{m})$ is analytically unramified. An example due to Cutkosky-Hà-Srinivasan-Theodorescu shows that this limit can be an irrational number even in a regular local ring. Throughout this talk, we shall restrict ourselves to homogeneous ideals in a standard graded domain over an algebraically closed field of arbitrary characteristic. Inspired by Trivedi's approach to Hilbert-Kunz multiplicity via density functions, we shall introduce a real valued compactly supported continuous function whose integral gives the ε-multiplicity. If time permits, we shall produce some explicit examples.

PROJECTIVE MODULES, CLASSICAL K-THEORY AND LINEAR ALGEBRA

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In 1955-56, a French mathematician J-P. Serre asked if finitely generated projective modules over polynomial rings over fields are free. In 1976 D. Quillen and A. Suslin independently proved it affirmatively. Soon after, following Quillen's idea, Suslin gave a new proof using results in matrix theory. In this talk we will discuss the related problems in linear algebra and their generalizations for classical groups.

AUSLANDER-REITEN CONJECTURE AND FINITE HOMOLOGICAL DIMENSION OF HOM

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One of the most remarkable long-standing conjectures in homological commutative algebra is due to Auslander and Reiten: A finitely generated module M over a commutative Noetherian ring R is projective if $\mathrm{Ext}^n_R(M, M \oplus R) = 0$ for all $n \geq 1$. We show that the conjecture is affirmative in each of the following cases: (1) $\mathrm{id}_R(M^*)$ is finite, (2) $\mathrm{id}_R(\mathrm{Hom}_R(M,M))$ is finite, (3) $\mathrm{pd}_R(M^*)$ is finite, (4) $\text{G-dim}_R(M)$ and $\mathrm{pd}_R(\mathrm{Hom}_R(M,M))$ are finite, where $M^* := \mathrm{Hom}_R(M,R)$, and id, pd and G-dim denote the injective, projective and Gorenstein dimensions respectively.

These results are the main contents of two articles [1] and [2] joint with Souvik Dey and Ryo Takahashi respectively.

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MONIC INVERSION PRINCIPLE AND COMPLETE INTERSECTION

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The monic inversion principle is a recurring theme in the area of projective modules and complete intersection ideals over polynomial rings.It has been started to steal the limelight from the early 80's, when Quillen used monic inversiion principle for the freeness of projective modules to solve the celebrated Serre's conjecture. I shall start our discussion with few known monic inversion principles from literature.

Then we will discuss our main result which is the monic inversion principle for the projective generation of an ideal. An ideal I is said to be projectively generated if there exists a surjection $\alpha : P \to I$, where rank of projective module P is the same as the height of I.

Let A be a regular ring of dimension d essentially of finite type over an infinite field k of characteristic $\neq 2$. Let P be a projective A-module of rank n with $2n \geq d+3$. Let I be an ideal of $A[T]$ of height n and $\phi : P[T] \to I/I^2$ be a surjection. If ϕ_f has a surjective lift $\theta : P[T]_f \to I_f$, for some monic polynomial $f \in A[T]$ then ϕ has a surjective lift $\Phi : P[T] \to I$.

If time permits, I will emphasize the methods of our proof and the key results which were crucial for the proof.