MATINGS, HOLOMORPHIC CORRESPONDENCES, AND A BERS SLICE

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Combination theorems have a long and rich history in geometry and dynamics. There are two frameworks for mating/combining Kleinian groups with rational maps on the Riemann sphere: an algebraic correspondence framework due to Bullett-Penrose-Lomonaco and an orbit equivalence mating framework using Bowen-Series maps. The latter is analogous to the Douady-Hubbard theory for polynomial mating. We will discuss how these two frameworks can be unified and generalized. Using this recipe, we will construct holomorphic correspondences that are matings of hyperbolic orbifolds (including Hecke surfaces and punctured spheres) with polynomials, and describe complex-analytic embeddings of Teichmüller spaces of the above orbifolds in the algebraic parameter space of correspondences.

Based on joint work with Mahan Mj.

MODEL SPACES INVARIANT UNDER COMPOSITION OPERATORS

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In this talk, first we discuss about a connection between the famous invariant subspace problem and composition operators induced by Möbius transformations. Next, we discuss common invariant subspaces of shift and composition operators. Finally, we move to model spaces invariant under composition operators and some open problems in this direction.

UNIFORM APPROXIMATION ON POLYNOMIAL POLYHEDRA

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Recently Samuelsson and Wold proved that if $f_j \in \mathcal{C}(\partial\mathbb{D} \times \partial\mathbb{D})$, $j = 1, \ldots, N$, such that each f_j extends to pluriharmonic function on $\mathbb{D} \times \mathbb{D}$, then either the uniform algebra generated by $z_1, z_2, f_1, \ldots, f_N$ is equal to all continuous functions on $\partial\mathbb{D} \times \partial\mathbb{D}$ or there exists a nontrivial algebraic variety Z in $\mathbb{D} \times \mathbb{D}$ such that each f_j is holomorphic in Z. We will discuss the analogous result for certain polynomial polyhedra where things are much more subtle.

GEOMETRY OF THE CASSINIAN METRIC

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This talk considers the Cassinian metric which is defined on a proper subdomain of the n-space $\mathbb{R}^n$ ($n \geq 2$). This metric was introduced in 2009. Indeed, for a pair of points x, y lying on a domain $D \subsetneq \mathbb{R}^n$, the Cassinian metric c_D is defined by

$$c_D(x, y) = \sup_{p \in \partial D} \frac{|x - y|}{|x - p|\,|y - p|},$$

where ∂D stands for the boundary of D. There are two techniques to build this metric: (i) taking maximal Cassinian ovals into account; and (ii) measuring the Euclidean distance between inversions of x, y in the unit sphere centered at the point p. The Cassinian ovals are helpful in establishing the Earth's orbit and modeling population growth in the actual world. For the audience's convenience, some Cassinian metric geometries are covered in the first steps. This metric is not typically either quasiconformal or Möbius invariant. Our final attention is on the geometry of this and its scale invariant metrics under Möbius and quasiconformal mappings.

THE ENTROPY OF A CERTAIN SEMIGROUP OF HOLOMORPHIC MAPS

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In 2008, Dinh and Sibony introduced the notion of topological entropy for meromorphic correspondences defined on a compact Kähler manifold and obtained an upper bound for the same. In this lecture, we study a class of holomorphic correspondences (namely a finitely generated rational semigroup of holomorphic maps) whose entropy equals the known upper bound.

This is a joint work with G. Bharali.

GAP THEOREMS IN BICOMPLEX SPACE

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In the region of convergence, a power series converges. Within its circle of convergence, it is completely convergent, but it may behave differently outside of it. The behaviour of the power series' boundaries on the circle of convergence of the bicomplex space is discussed in this article. The bicomplex Hadamard's gap theorem and a few other theorems have been proven for the power series in bicomplex space. The unit hypersphere in the bicomplex space is also shown. The Vivanti-Pringsheim theorem and its extension have been demonstrated for series in bicomplex space.

LIMITS OF AN INCREASING SEQUENCE OF COMPLEX MANIFOLDS

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The purpose of this talk is to provide an overview of a recent joint work on a version of the 'union problem' with D. Borah, P. Mahajan and K. Verma. The 'union problem' consists of the following. Let M be a complex manifold which admits an exhaustion by open subsets M_j each of which is biholomorphic to a fixed domain $\Omega \subset \mathbb{C}^n$; describe M in terms of Ω. Building on work of Fornaess–Sibony, we studied two cases namely, M is Kobayashi hyperbolic and the other being the corank one case in which the Kobayashi metric degenerates along one direction. When M is Kobayashi hyperbolic, its complete description is obtained when Ω is one of the following domains: (i) a smoothly bounded Levi corank one domain, (ii) a smoothly bounded convex domain, (iii) a strongly pseudoconvex polyhedral domain in $\mathbb{C}^2$, or (iv) a simply connected domain in $\mathbb{C}^2$ with generic piecewise smooth Levi-flat boundary. With additional hypotheses, the case when Ω is the minimal ball or the symmetrized polydisc in $\mathbb{C}^n$ can also be handled. After a quick overview of this in the first half of the talk, we shall move on to the question of addressing a degenerate case, as dealt with in the above-mentioned work. Namely, when the Kobayashi metric on M has corank one and Ω is either of (i), (ii) or (iii) listed above, it is shown that M is biholomorphic to a locally trivial fibre bundle with fibre $\mathbb{C}$ over a holomorphic retract of Ω or that of a limiting domain associated with it. Finally, when $\Omega = \Delta \times \mathbb{B}^{n-1}$, the product of the unit disc $\Delta \subset \mathbb{C}$ and the unit ball $\mathbb{B}^{n-1} \subset \mathbb{C}^{n-1}$, a complete description of holomorphic retracts is obtained. As a consequence, if M is Kobayashi hyperbolic and $\Omega = \Delta \times \mathbb{B}^{n-1}$, it is shown that M is biholomorphic to Ω. Further, if the Kobayashi metric on M has corank one, then M is globally a product; in fact, it is biholomorphic to $Z \times \mathbb{C}$, where $Z \subset \Omega = \Delta \times \mathbb{B}^{n-1}$ is a holomorphic retract.

SOME REMARKS ON THE SQUEEZING FUNCTION

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The squeezing function is a biholomorphic invariant of bounded domains introduced by Deng, Guan, and Zhang and is a measure of how well a domain can be squeezed between two Euclidean balls. In this talk, we will discuss a class of problems associated with its computation and connections with the intrinsic geometry of domains. We will also present a new application of this function.

This is joint work with Gautam Bharali and Sushil Gorai.

AREA THEOREMS FOR CERTAIN CLASSES OF FUNCTIONS HAVING QUASICONFORMAL EXTENSIONS

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Let $\Sigma_k(p)$ and $\Sigma_H^k(p)$ be the classes of all meromorphic univalent functions and sense-preserving univalent harmonic mappings, respectively, defined in the open unit disc of the complex plane with a simple pole at $z = p \in (0, 1)$, that have k-quasiconformal $(0 \leq k < 1)$ extensions to the extended complex plane. In this talk, we will discuss about area theorems for these classes of functions.

THE FRIEDRICHS OPERATOR AND CIRCULAR DOMAINS

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The Friedrichs operator of a domain (in $\mathbb{C}^n$) is closely related to its Bergman projection and encodes crucial information (geometric, quadrature, potential theoretic etc.) about the domain. In this talk, we will show that the Friedrichs operator of a domain has rank one if the domain can be covered by a circular domain via a proper holomorphic map of finite multiplicity whose Jacobian is a homogeneous polynomial. As an application, we show that the Friedrichs operator is of rank one on the tetrablock, pentablock, and the symmetrized polydisc – domains of interest in the study of μ-synthesis in control theory.

This is joint work with Samriddho Roy, ISI Delhi.

BAKER DOMAINS IN THE DYNAMICS OF MEROMORPHIC FUNCTIONS

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The dynamics of transcendental meromorphic functions differ in many ways from the dynamics of rational functions. The Fatou set $\mathcal{F}(f)$ of a (nonconstant and nonlinear) meromorphic function f is the subset of the extended complex plane $\widehat{\mathbb{C}}$ where the sequence of iterates $\{f^n\}$ of f is defined and form a normal family. A connected component of $\mathcal{F}(f)$ is called a Fatou component. A periodic Fatou component U of period p is called a Baker domain if there exists $z_0 \in \partial U$ such that $\{f^{np}(z)\}$ converges to z_0 for all $z \in U$ as $n \to \infty$, but $f^p(z_0)$ is not defined. It is well known that the Fatou set of an entire function in the Speiser class does not contain Baker domains. In this talk, we will present some of the recent developments on the Baker domains in the dynamics of transcendental meromorphic functions.

ON STARLIKENESS OF REGULAR COULOMB WAVE FUNCTIONS

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In this talk, I will present some geometric properties of a class of analytic functions which is defined from the J-fraction expansion of the ratio $zf'(z)/f(z)$. I will find the disk domain which is mapped into a starlike domain by these functions. Moreover, I will present similar results for two different normalized forms of regular Coulomb wave functions and a normalized Bessel function of the first kind by using continued fractions expansions.

BOHR RADIUS FOR BANACH SPACES ON SIMPLY CONNECTED DOMAINS

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Let $H^\infty(\Omega, X)$ be the space of bounded analytic functions $f(z) = \sum_{n=0}^{\infty} x_n z^n$ from a proper simply connected domain Ω containing the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ into a complex Banach space X with $\|f\|_{H^\infty(\Omega,X)} \leq 1$. In 1914, Harald Bohr published the following remarkable result: Let $f(z) = \sum_{n=0}^{\infty} a_n z^n \in H^\infty(\mathbb{D}, \mathbb{C})$. Then, $\sum_{n=0}^{\infty} |a_n z^n| \leq \|f\|_\infty$ when $|z| \leq 1/3$, and moreover the constant $1/3$, called the *Bohr radius* is optimal. Let $\phi = \{\phi_n(r)\}_{n=0}^{\infty}$ with $\phi_0(r) \leq 1$ such that $\sum_{n=0}^{\infty} \phi_n(r)$ converges locally uniformly with respect to $r \in [0, 1)$. For $1 \leq p, q < \infty$, we denote

$$R_{p,q,\phi}(f, \Omega, X) = \sup \left\{ r \geq 0 : \|x_0\|^p \phi_0(r) + \left(\sum_{n=1}^{\infty} \|x_n\| \phi_n(r) \right)^q \leq \phi_0(r) \right\}$$

and define the *Bohr radius* associated with ϕ by

$$R_{p,q,\phi}(\Omega, X) = \inf \left\{ R_{p,q,\phi}(f, \Omega, X) : \|f\|_{H^\infty(\Omega,X)} \leq 1 \right\}.$$

In this talk, we extensively discuss the Bohr radius $R_{p,q,\phi}(\Omega, X)$, when X is an arbitrary Banach space and $X = \mathcal{B}(\mathcal{H})$ is the algebra of all bounded

linear operators on a complex Hilbert space $\mathcal{H}$. Furthermore, we present the Bohr inequality for the operator-valued Cesáro operator and Bernardi operator.

A SURVEY ON THE EXISTENCE AND FORMS OF SOLUTIONS OF DIFFERENT VARIANTS OF FERMAT TYPE FUNCTIONAL EQUATIONS

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Fermat type functional equation in $\mathbb{C}$

$$f^n(z) + g^n(z) = 1 \qquad (1.1)$$

is generated from the classical Fermat Diophantine equation $x^n + y^n = 1$, where n is any positive integer. For $n > 2$, Montel [3] established that equation (1.1) has no entire solution. The case $n = 2$ was settled by Gross [1] by finding the exact form of entire solution of $f^2(z) + g^2(z) = 1$ as $f(z) = cos(h(z))$ and $g(z) = sin(h(z))$, $h(z)$ is an entire function. Further for $n = 2$, the meromorphic solutions of (1.1) was also investigated by Gross [2] as $f(z) = \frac{2\alpha(z)}{1+\alpha(z)^2}$ and $g(z) = \frac{1-\alpha(z)^2}{1+\alpha(z)^2}$, where $\alpha(z)$ is a non-constant meromorphic function in $\mathbb{C}$.

With the development of difference analogue of the Nevanlinna theory, many authors contributed significantly to the study of existence and forms of solutions of several variants of Fermat type functional equations generated by several operators like shift, difference, difference-differential etc.

During the last few years there were several generalizations and refinements of the existing results in terms of binomial, trinomial equations, partial differential equations, differential difference equations, system of general equations etc. In this survey, we discuss about the various forms of solutions of different quadratic binomials and trinomials generated by shift or difference operators, differential-difference operators on $\mathbb{C}$ and higher dimension.

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AREA-MINIMIZING MINIMAL GRAPHS OVER NON-CONVEX DOMAINS

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The well-established notion is that minimal surfaces over convex domains exhibit a global area-minimizing property, a characteristic not necessarily shared by their counterparts over non-convex domains. In an article titled "Area-minimizing minimal graphs over non-convex domains," published in Pacific J. Math., M. Dorff, D. Halverson, and G. Lawlor demonstrated that minimal surfaces over a bounded linearly accessible domain D of order β (where β lies in the range $(0, 1)$), must be globally area-minimizing under specific geometric conditions satisfied on the boundary of D.

In this presentation, we discuss sufficient conditions for a sense-preserving harmonic function $f = h+\overline{g}$ to be linearly accessible of order β, with β falling within the interval $[0, 1]$. Additionally, we explore a method for constructing harmonic polynomials using the Gaussian hypergeometric function. This construction ensures that the range of these polynomials forms a linearly accessible domain of order β. Leveraging these harmonic polynomials, we establish one-parameter families of globally area-minimizing minimal surfaces over non-convex domains. Furthermore, we investigate results related to approximating a minimal surface through a sequence of bounded minimal surfaces.

ON ROTATIONAL DOMAINS FOR MEROMORPHIC FUNCTIONS

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While the field of complex dynamics is full of many interesting objects, there are particularly two which stands out due to their complex nature. These two are Siegel disks and Herman rings. Our work focus on these periodic Fatou components. First a discussion on the various possible arrangements of 3-cycle and 4-cycle Herman rings of transcendental meromorphic functions having at least one omitted value ($\mathcal{M}_0$) is given. We show that for $p = 3, 4, 5$, the number of p-cycle Herman rings can be at most one. It is also shown that a 3-cycle and a 4-cycle Herman ring can not exist simultaneously. Further we investigate the configurations of Herman rings for the class $\mathcal{M}_0$. It is proved that the possible number of configurations of a 5-periodic Herman ring of an element of $\mathcal{M}_0$ is 6. Further, we present a result about the non-existence of a 3-periodic Herman ring and a 5-periodic Herman ring simultaneously. Some transcendental meromorphic functions having no Herman ring are discussed. We also study the existence of Siegel disks for the functions which are meromorphic outside a small set, called the class $\mathcal{K}$. We provide some results which are helpful to find examples of functions having invariant Siegel disks. Non-existence of Siegel disk for some classes is also discussed. Finally, we provide some images of Siegel disks verifying our results.