THE RMS2023 SYMPOSIUM ON "GROUPS AND THEIR REPRESENTATIONS"

TITLES AND ABSTRACTS

(ST-1) Speaker: C. S. Rajan

Title: Relative Weyl Character formula, Relative Pieri formulas and Branching rules for Classical groups

Abstract: We give alternate proofs of the classical branching rules for highest weight representations of a complex reductive group G restricted to a closed regular reductive subgroup H, where (G, H) consist of the pairs (GL(n+1), GL(n)), (Spin(2n+1), Spin(2n)) and (Sp(2n), Sp(2)xSp(2n-2)). Our proof is essentially a long division. The starting point is a relative Weyl character formula and our method is an inductive application of a relative Pieri formula. We also give a proof of the branching rule for the case of (Spin(2n), Spin(2n-1)), by a reduction to the case of (GL(n), GL(n-1)).

This is joint work with Sagar Shrivastava.

(ST-2) Speaker: R. Venkatesh

Title: Structure of symmetric regular subalgebras of affine Kac–Moody algebras

Abstract: Let $\mathfrak{g}$ be a finite dimensional semi-simple Lie algebra with a Cartan subalgebra $\mathfrak{h}$ and let Φ be the corresponding root system. Any $\mathfrak{h}$-invariant subalgebra is called a regular subalgebra. In one of his influential papers, E. B. Dynkin (1952) established the one-to-one correspondence between regular semi-simple subalgebras of $\mathfrak{g}$ and the closed subroot systems of Φ, and using this he classified all semi-simple subalgebras of $\mathfrak{g}$. Note that any symmetric closed subset of Φ must be a closed subroot system. This is not, in general, true even for real affine root systems.

Motivated from Dynkin's work, we study the subalgebras of affine Kac-Moody algebras, called symmetric regular subalgebras that are generated by symmetric closed subsets. In this talk, I will explain the structure of these algebras. Indeed we prove that, these algebras are semi-direct products of affine and Heisenberg type algebras. This is a joint work with Irfan Habib and Deniz Kus.

(ST-3) Speaker: Ravi Raghunathan

Title: Pairs of L-functions of symmetric powers of cuspidal automorphic representations

Abstract: If π is a cuspidal automorphic representation of $GL_2/\mathbb{Q}$ the associated symmetric power L-functions $L(s, sym^n, \pi)$ contain important arithmetic information. We will prove a theorem about the zero sets of pairs of such L-functions. Time permitting, we will introduce the notion (due to Dipendra Prasad) of immersion of one automorphic representation in another and mention some preliminary results (proved jointly with Dipendra Prasad) characterising pairs of representations such that one is immersed in the other.

(ST-4) Speaker: Pooja Singla

Title: Constructing Continuous Complex Irreducible Representations of $\mathrm{GL}_3(\mathfrak{O})$

Abstract: Let $\mathfrak{O}$ be a non-Archimedean local field with a finite residue field of characteristic p. In this lecture, we delve into the construction of continuous complex irreducible representations of the general linear groups $\mathrm{GL}_3(\mathfrak{O})$ while focusing on their representation growth. Specifically, we address the case where $\mathfrak{O}$ has a positive characteristic. The corresponding results for zero characteristic $\mathfrak{O}$ are due to Nir Avni, Benjamin Klopsch, Uri Onn, and Christopher Voll. We assume that $p > 3$ and aim to provide a unified construction that applies to all $\mathfrak{O}$. This is based on joint work with Uri Onn and Amritanshu Prasad.

(ST-5) Speaker: Manoj K. Yadav

Title: Wells' fundamental exact sequence: Then and Now

Abstract: In 1971, Charles Wells invented a 4-term fundamental exact sequence relating (lifting and extension of) automorphisms and second cohomology group of group extensions. Since then the sequence has been extensively studied and reformulated by various mathematicians in many different contexts. It is planned to exhibit Wells' exact sequence for various algebraic structures, namely, groups, Lie algebras, Lie rings, quandles, cycle sets, skew braces and others.

(ST-6) Speaker: Basudev Pattanayak

Title: Fixed vectors under principal congruence subgroups of $\mathrm{GL}(n)$

Abstract: Let G be an inner form of a general linear group over a non-archimedean local field. The goal of this talk is to give a necessary and sufficient condition for the existence of a non-trivial fixed vector of an irreducible smooth representation of G under principal congruence subgroups in terms of conductor and Moy-Prasad depth of the representation. As an application, we explicitly compute the dimension of fixed vectors of a principal series representation under the congruence subgroup.

(ST-7) Speaker: Kumar Balasubramanian

Title: Twisted Jacquet Module of a cuspidal representation of GL($2n$)

Abstract: Let F be a finite field and $G = \mathrm{GL}(2n, F)$. We will present some results on the structure of the twisted Jacquet module for a cuspidal representation of G.

(ST-8) Speaker: Arghya Mondal

Title: Asymptotic Shur orthogonality relations

Abstract: Schur's orthogonality relations are between matrix coefficients of a compact group G in $L^2(G)$. One may try to remedy the failure of matrix coefficients of a non compact group to be in L^2 by averaging over a sequence of finite measure subsets F_n and taking limit. Such modified relations, when they exist, are called *asymptotic Schur orthogonality relations.* They exist for second countable locally compact abelian groups, where $\{F_n\}_n$ can be any Følner sequence. Their existence is also conjectured and verified in some cases, by Kazhdan and Yom Din, for tempered representations of semisimple groups over local fields. We will discuss the existence of such relations for the Heisenberg group.

(ST-9) Speaker: Pralay Chatterjee

Title: Lower dimensional Betti numbers of homogeneous spaces of Lie groups

Abstract: We will present our joint work with I. Biswas and C. Maity on certain explicit descriptions of lower dimensional Betti numbers of homogeneous spaces of Lie groups. We will begin by recalling some of the earlier works in this subject and give our motivation. We will then describe our results and some of the applications in special cases. If time permits we will also sketch some of the proofs.

(ST-10) Speaker: Mahendra Verma

Title: The theory of Symplectic models

Abstract: For a subgroup H of a group G, we say a representation V of G is said to be H-distinguished if there exists a non-zero H-invariant linear form on V. We will discuss distinguished representations for $G = GL(n, D)$ and $H = Sp(n, D)$ where D is either a non-archimedean local field k or the quaternion division algebra over k and in this case, we say such representations have symplectic models. This is a joint work with Hariom Sharma.

(ST-11) Speaker: U. K. Anandavardhanan

Title: The correlation coefficient in representation theory

Abstract: Given a group G and two Gelfand subgroups H and K of G, associated to an irreducible representation π of G, there is a notion of H and K being correlated with respect to π in G. This notion was defined by Benedict Gross in 1991. We discuss this theme and give some details in some specific examples (which are part of joint works with Arindam Jana and Basudev Pattanayak).

(ST-12) Speaker: Amiya Mondal

Title: On the bounds of Swan conductor of functorial lifts

Abstract: The Swan conductor of a smooth, complex, semisimple local Galois representation is an additive invariant of the representation. In this talk, we will describe the best possible upper bounds of the Swan conductor of the following functorial lifts: symmetric square, exterior square and Asai lift. This is part of a work in progress with Arindam Jana.

(ST-13) Speaker: Shashank Vikram Singh

Title: Some Arithmetic Symplectic Hypergeometric Groups

Abstract: A hypergeometric group $\Gamma(f,g)$ is a subgroup of the general linear group generated by the companion matrices of some co-prime polynomials f,g. Under appropriate assumptions on the polynomials f,g, $\Gamma(f,g)$ preserves a non-degenerated symplectic form Ω and $\Gamma(f,g) \subseteq \mathrm{Sp}_\Omega(\mathbb{Z})$. If $[\mathrm{Sp}_\Omega(\mathbb{Z}) : \Gamma(f,g)] < \infty$, then $\Gamma(f,g)$ is an arithmetic symplectic hypergeometric groups. In this talk, we will discuss a criterion that helps in determining the arithmeticity of the hypergeometric group $\Gamma(f,g)$.

(ST-14) Speaker: Arunava Mandal

Title: Universal subspaces for Lie groups

Abstract: Let U be a finite dimensional vector space over $\mathbb{R}$ or $\mathbb{C}$, and let $\rho : G \to GL(U)$ be a representation of a connected Lie group G. A linear subspace $V \subset U$ is called universal if every orbit of G meets V. In this talk, we study universal subspaces for Lie groups, especially compact Lie groups. This is a joint work with Saurav Bhaumik.

(ST-15) Speaker: Shiv Parsad

Title: On two generator subgroups of $\mathrm{SU}(\mathbf{3},\mathbf{1})$

Abstract: Let $A,\ B$ be two loxodromic elements in $\mathrm{SU}(3,1)$ such that they generate a 'non-singular' subgroup $\langle A,B\rangle$. In this talk, we show that $\langle A,B\rangle$ is determined up to conjugacy by the following parameters:

$$tr(A),\ tr(B),\ \sigma(A),\ \sigma(B),\ \mathbb{X}_k(A,B),\ k=1,2,3, \text{ one } \alpha\text{-invariant and one } \beta\text{-invariant},$$

where $tr(A) = \mathrm{trace}(A)$, $\sigma(A) = \frac{1}{2}(tr^2(A) - tr(A^2))$, $\mathbb{X}_k(A,B)$, α-invariant and β-invariant are defined by eigenvectors of A and B and the Hermitian form. This is a joint work with Krishnendu Gongopadhyay.