Saikatul Haque

Title

Ill-posedness of Hartee Equation

Abstract

D G Bhimani and Saikatul Haque

In this talk, I planned to present ill-posedness in Fourier amalgam spaces for Hartree Equation (which is a nonlinear Schrödinger equation (NLS) with convolution type nonlinearity). We establish norm inflation (stronger phenomena than mere ill-posedness) for this NLS in Fourier amalgam spaces with negative regularity. We further show that this can be even worse by exhibiting norm inflation with infinite loss of regularity. It turns out that our approach could treat the NLS with both the nonlinearities (power-type and Hartree-type) in a unified manner.

Ved Datar

Title

Non-linear PDEs on Kahler manifolds and positivity conditions

Abstract

It is well known that solvability of the complex Monge-Ampere equation on compact Kaehler manifolds is related to the positivity of certain intersection numbers. In fact, this follows from combining Yau's celebrated resolution of the Calabi conjecture, with Demailly and Paun's generalization of the classical Nakai-Mozhesoin criteria. This correspondence was recently extended to a broad class of complex non-linear PDEs including the J-equation and the deformed Hermitian-Yang-Mills (dHYM) equations by the work of Gao Chen and others (including some at IISc). I will survey some of these recent developments.

Debanjana Mitra

Title

Lack of null controllability of certain linear transport-parabolic coupled systems

Abstract

In my talk, I shall discuss the control aspects of linear coupled transport-parabolic systems with variable coefficients in one space dimension. The systems are considered with homogeneous Dirichlet boundary conditions and with localized interior controls acting on both transport and parabolic equations. We show that coupled systems are not null controllable at small time. This time depends on the transport velocity and the support of the controls. When the transport velocity is identically zero, the systems are not null controllable at any time. To achieve these results, we construct highly localized solutions, known as *Gaussian beams*, corresponding to the adjoint systems, and using them, we show that the corresponding observability inqualities

fail. However, these systems are null controllable at any time by controls acting everywhere in the parabolic equation, under suitable assumptions on the initial data and the coefficients.

This a joint work with S. Ahamed and D. Maity published as 'Lack of null controllability of one dimensional linear coupled transport-parabolic system with variable coefficients, Journal of Differential Equations (2022), Vol. 320, pp. 64-113.

Saikat Mazumdar

Title

Existence results for the higher-order Q-curvature equation

Abstract

In this talk, we will obtain some existence results for the Q-curvature equation of arbitrary $2k$-th order, where $k \geq 1$ is an integer, on a compact Riemannian manifold of dimension $n \geq 2k+1$. This amounts to solving a nonlinear elliptic PDE involving the powers of Laplacian called the GJMS operator. The difficulty in determining the explicit form of this GJMS operator together with a lack of maximum principle complicates the issues of existence.
This is a joint work with Jerome Vetois (McGill University).

Kartick Ghosh

Title

Coupled Kähler-Einstein AND Hermitian-Yang-Mills equations

Abstract

We shall discuss coupled Kähler-Einstein and Hermitian-Yang-Mills equations. These are a system of partial differential equations. These equations have an interpretation in terms of a moment map. We will show that there are some obstructions to the existence of solutions, namely, Matsushima-Lichnerowicz-type reductivity and Futaki-type invariant. We then solve the equations in some non-trivial cases using deformation. We will also produce a solution of these equations on some projective bundles.

Alok Kumar Sahoo

Title

Global compactness result and multiplicity of solutions for a class of critical exponent problem in the hyperbolic space

Abstract

Consider the problems of the type

$$-\Delta_{\mathbb{B}^N} u - \lambda u = a(x)|u|^{2^*-2}u + f(x) \quad \text{in } \mathbb{B}^N, \quad u \in H^1\left(\mathbb{B}^N\right),$$

where $\mathbb{B}^N$ denotes the ball model of the hyperbolic space of dimension $N \geq 4, 2^* = \frac{2N}{N-2}$, $\frac{N(N-2)}{4} < \lambda < \frac{(N-1)^2}{4}$ and $f \in H^{-1}\left(\mathbb{B}^N\right)$ $(f \not\equiv 0)$ is a non-negative functional in the dual space of $H^1\left(\mathbb{B}^N\right)$. The potential $a \in L^\infty\left(\mathbb{B}^N\right)$ is assumed to be strictly positive, such that $\lim_{d(x,0)\to\infty} a(x) = 1$, where $d(x,0)$ denotes the geodesic distance. For $f = 0$ and $a \equiv 1$, profile decomposition was studied by Bhakta and Sandeep [Calc. Var. PDE, 2012]. However, due to the presence of the potential $a(.)$, an extension of profile decomposition to the present set-up is highly nontrivial and requires several delicate estimates and geometric arguments concerning the isometry group (Möbius group) of the hyperbolic space. Our result generalizes the profile decomposition. Further, using the decomposition result, we derive various energy estimates involving the interacting hyperbolic bubbles and hyperbolic bubbles with localized Aubin-Talenti bubbles. Finally, combining these estimates with topological and variational arguments, we establish a multiplicity of positive solutions in the cases: $a \geq 1$ and $a < 1$ separately. The equation studied in this article can be thought of as a variant of a scalar-field equation with a critical exponent in the hyperbolic space, although such a critical exponent problem in the Euclidean space $\mathbb{R}^N$ has only a trivial solution when $f \equiv 0, a(x) \equiv 1$ and $\lambda < 0$.

This is a Joint work with Mousomi Bhakta, Debdip Ganguly and Diksha Gupta.

Aashirwad Naveen Ballal

Title

The supercritical deformed Hermitian Yang-Mills equation on compact projective manifolds

Abstract

We discuss a result regarding the solvability of the “twisted deformed Hermitian Yang-Mills equations" on compact Kähler manifolds with a slightly negative twisting function. Using this result, we prove that the twisted dHYM equation on compact, projective manifolds can be solved provided certain numerical conditions are satisfied. As a corollary, we obtain a new proof in the projective case of a recent theorem addressing a conjecture of Collins-Jacob-Yau.

Debdip Ganguly

Title

Sharp quantitative stability of Poincaré-Sobolev inequality in the hyperbolic space

Abstract

The talk is devoted to the sharp stability of Poincaré-Sobolev inequalities in the hyperbolic space. To begin with, I shall formulate the question of the stability of the classical Sobolev inequality in the Euclidean space and recall some of the seminal results of Bianchi-Egnell and Ciraolo, Figalli and Maggi and many others. Then I shall deduce the (sharp) quantitative gradient stability of the Poincaré-Sobolev inequalities in the hyperbolic space and the corresponding Euler-Lagrange equation locally around a bubble (and possibly at a higher energy level!) if time permits. This is joint work with M. Bhakta, D Karmakar, and S. Mazumdar.

Sheela Verma

Title

Shape optimization problem for Steklov-Dirichlet eigenvalues

Abstract

Sagar Basak, Anisa Chorwadwala, Sheela Verma

Let Ω be a bounded smooth domain in $\mathbb{R}^n$ with two disjoint boundary components C_1 and C_2. The mixed Steklov-Dirichlet problem is to find harmonic function u in Ω such that $u = 0$ on C_1 and outer normal derivative of u is directly proportional to u along C_2. This problem models the stationary heat distribution in Ω with the conditions that the temperature along C_1 is kept to zero and that the heat flux through C_2 is proportional to the temperature. In this talk, I will first discuss about behaviour of the first Steklov Dirichlet eigenvalue on doubly connected domains and then provide some isoperimetric bounds for higher Steklov Dirichlet eigenvalues.

Sarika Goyal

Title

Quasilinear Schrödinger Equations Involving Choquard Type Critical Nonlinearity

Abstract

Sarika Goyal, Reshmi Biswas and K. Sreenadh

In this talk, we will discuss about the existence results of quasilinear Schrödinger equation involving critical Choquard type exponential nonlinearity in bounded domain as well as in unbounded domain. In particular, we will discuss the existence results of following quasilinear Schrödinger equation

$$\left\{-\Delta_N u - \Delta_N\left(u^2\right)u + V(x)|u|^{N-2}u = \left(\int_{\mathbb{R}^N}\frac{F(y,u)}{|x-y|^\mu}dy\right)f(x,u) \quad \text{in } \mathbb{R}^N,\right.$$

where $N \geq 2, 0 < \mu < N$. The potential $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous function satisfying $0 < V_0 \leq V(x)$ for all $x \in \mathbb{R}^N$ and some suitable assumptions. The nonlinearity $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a continuous function with critical exponential growth in the sense of the Trudinger-Moser inequality and $F(x,s) = \int_0^s f(x,t)dt$ is the primitive of f.

Souptik Chakraborty

Title

Some quantitative Sobolev-type inequalities

Abstract

In this talk, we will consider some quantitative Sobolev-type inequalities where the optimal constant and extremal functions are known. We ask, if a function "almost attains the equality" then is it "quantitatively close to the manifold of extremal functions" in some appropriate sense? Next, shifting our attention from the inequality to the corresponding Euler-Lagrange equation, we analyze when a function that almost solves the equation is "quantitatively close to the manifold of the sum of critical points". These results are in the spirit of Bianchi-Egnell [1991], Ciraolo-Figalli-Maggi [2017], Figalli-Glaudo [2020], and Deng-Sun-Wei (Pre-print).

Abhrojyoti Sen

Title

Fine boundary regularity for fully nonlinear mixed local-nonlocal problems

Abstract

In this talk, we will consider Dirichlet problems for fully nonlinear mixed local-nonlocal non-translation invariant operators. For a bounded C^2 domain $\Omega \subset \mathbb{R}^d$, let $u \in C(\mathbb{R}^d)$ be a viscosity solution of such Dirichlet problem. We discuss the global Lipschitz regularity and fine boundary regularity for u by constructing appropriate sub and supersolutions coupled with a "weak version" of Harnack inequality. Moreover, applying these results we will discuss the Hölder regularity of Du up to the boundary. We conclude the talk with some ongoing and future works.

Subrata Majumdar

Title

Controllability of the linearized compressible Navier-Stokes system with Maxwell's law (Sakil Ahamed and Subrata Majumdar)

Abstract

In this talk, we discuss the control properties of the linearized compressible Navier-Stokes system with Maxwell's law around a constant steady state $(\rho_s, u_s, 0)\,, \rho_s > 0, u_s > 0$ in the interval $(0, 2\pi)$ with periodic boundary data. We explore the exact controllability of the coupled system by means of a localized interior control acting in any of the equations when time is large enough. We study the exact controllability of the system in the space $L^2(0, 2\pi) \times L^2(0, 2\pi) \times L^2(0, 2\pi)$ by proving an observability inequality with the help of an Ingham-type inequality. Next, we discuss some small time lack of controllability of the concerned system for the case of localized interior control.