

# DISTANCES IN SETS

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## Abstract

Finding patterns in the given set is a significant interest in the recent decades. One of the simple patterns is distances, namely finding two term progression in the given set is one of the examples. The distance set of a set  $E$  consists of all non-negative numbers that represent distances between pairs of points in  $E$ . In this talk, we will review a few research problems in the history and discuss the research done in the recent years that approach solutions to finding distances of sets of positive measure and the structure of distance sets.

# Some special kinds of reversible and symmetric rings

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## Abstract

A ring  $R$  is defined to be quasi tri reversible if any non-zero tripotent element  $ab$  of  $R$  for  $a, b \in R$  implies  $ba$  is a tripotent element in  $R$ . We explore the quasi tri reversibility of 2 by 2 full and upper triangular matrix rings over various kinds of reversible rings. It is also observed that the structure of quasi tri reversible rings and weakly tri normal rings are independent of each other. Secondly, we introduce the notions of  $p$ -symmetric rings. We prove that  $R$  is a  $p$ -symmetric ring if and only if  $p^{n-1}Rp^{n-1}$  is a symmetric ring and  $p^{n-1}$  is left semi-central. Moreover we characterize  $p$ -symmetric rings in terms of upper triangular matrix rings and left min- $p$ -abel rings.

**Keywords:** Reversible rings, quasi tri reversible rings,  $p$ -symmetric rings

# TAMPERED RANDOM VARIABLE MODELING FOR MULTIPLE STEP-STRESS LIFE TEST

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In this paper, we introduce the Tampered Random Variable (TRV) modeling in multiple step-stress life testing experiments. Here  $\tau_1 < \tau_2 < \dots < \tau_{k-1}$  be  $(k-1)$  prespecified time points and  $s_1, s_2, \dots, s_k$  be  $k$  prefixed stress levels with  $s_i$  being the stress level in force during the time interval  $[\tau_{i-1}, \tau_i)$  for  $i = 1, \dots, k$  with  $\tau_0 = 0$  and  $\tau_k = \infty$ . We define the tampered random variable  $T_{TRV}^{(k)}$  in multi step-stress scenario and calculate the PDF, CDF, and Hazard rate for the proposed tampered variable  $T_{TRV}^{(k)}$ . We derive a general expression for the expectation of  $T_{TRV}^{(k)}$  under different number  $k$  of stress levels. We also prove some results on equivalence of the TRV modeling with the two other existing models for step-stress life testing, namely, Cumulative Exposure (CE) and Tampered Failure rate (TFR). Finally, we consider some variations of the modeling approach for  $T_{TRV}^{(k)}$  to suit some specific situations or purposes, including incorporation of the stress levels, discrete life time, bivariate or multivariate life times. Some inferencial work has been done in this direction.

# BAYESIAN VARIABLE SELECTION UNDER HIGH DIMENSIONAL SETTINGS WITH GROUPED COVARIATES

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Traditional Bayesian variable selection methods fail to yield satisfactory results in the normal linear regression setup when the number of covariates is much larger than the sample size, and the covariates form correlated groups. In such situations sparsity exists within and between groups, so that the response variable is not related to an entire group of covariates in all or none basis. We extend the Zellner's g-prior for regression parameters and the hierarchical uniform model prior to make them appropriate for this framework, and investigate the variable selection consistency property for the proposed method under fairly general conditions, assuming the covariates to be stochastic.

Implementation of the Bayesian variable selection methods in high dimensional scenarios is also challenging, even when the underlying method is consistent. The traditional stochastic search variable selection (SSVS) algorithms tend to get slow and require a high mixing time under high-dimensional correlated setups. We modify the existing SSVS algorithms using a class of group importance probabilities, termed as group-SIS (GSIS). Consequently, a novel stochastic search variable selection algorithm called group informed variable selection algorithm (GiVSA) is proposed, which uses the known group structure efficiently to explore the model space without discarding any of the covariates in an initial screening step. Performance of the proposed prior with implementation of GiVSA is discussed using a variety of numerical examples.

# FRACTAL DIMENSION AND FRACTIONAL CALCULUS OF NON-STATIONARY ZIPPER $\alpha$ -FRACTAL FUNCTIONS

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The fractal interpolation functions with appropriate iterated function systems (IFSs) provide a method to perturb and approximate a continuous function on a compact interval  $I$ . This method produces a class of functions  $f^\alpha$ , named as  $\alpha$ -fractal functions. As essential parameters of the IFS, the scaling factor  $\alpha$  has important consequences in the properties of the function  $f^\alpha$ . In this talk, we discuss the  $\alpha$ -fractal functions corresponding to the non-stationary zipper IFS. Here, we present a method to calculate an upper bound of the box and Hausdorff dimension of the proposed interpolant. Also, we provide an upper bound of the graph of the fractional integral of the proposed interpolant.

**A ROBUST NUMERICAL SOLVER FOR A BLOOD  
FLOW PROBLEM IN AN ELASTIC STENOTIC ARTERY  
IMPOSED WITH A GENERALIZED WOMERSLEY  
SOLUTION**

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The plaques due to stenosis can cause the narrowing of arteries, blocking the blood flow, which induces a time variation in the inlet mass flow rate and the related pressure drop. The existing numerical models in the literature to simulate the stenosis effect on the local blood flow have not accounted for the influence of moving boundaries on pressure near the arterial wall. In the present work, we develop a novel pressure-correction based numerical solver with coordinate transformation to simulate fluid flow in a moving stenotic artery, accounting for the generalized Womersley solution at the inlet. We obtain the pressure and velocity streamline contours in the physical domain using the inverse coordinate transformation to reveal the low-pressure and recirculation zones.

# DEMAZURE FILTRATIONS OF TENSOR PRODUCT MODULES AND CHARACTER FORMULA

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The current algebra  $\mathfrak{g}[t]$  of a simple Lie algebra  $\mathfrak{g}$  is the Lie algebra of polynomial maps from the field of complex number to  $\mathfrak{g}$ .

In the talk, I will define a Demazure module for the current Lie algebra associated to the Lie algebra of trace zero matrices and discuss our recent results on the structure of the tensor products of level 1-Demazure modules of  $\mathfrak{g}[t]$  with special Demazure modules. I will conclude the talk by giving the graded characters of these tensor product modules and relating them to certain partition identities. In particular, I will show that our result helps deduce explicit expressions for the product of a Schur polynomial with a specialized Macdonald polynomial in terms of Schur polynomials.

## OPTIMISATION OF A MIXED STEKLOV DIRICHLET EIGENVALUE

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**Abstract:** In this talk, I am going to talk about an eigenvalue optimisation problem over a family of doubly connected domains  $U := D \setminus \overline{\Omega}$  in  $\mathbb{R}^2$  where one boundary component,  $\partial D$ , is a circle while the other component  $\partial\Omega$  enjoys a dihedral symmetry. The Boundary Value Problem under consideration is  $\Delta u = 0$  on  $D \setminus \overline{\Omega}$ ,  $u = 0$  on  $\partial D$  and  $\frac{\partial u}{\partial n} = \sigma u$  on  $\partial\Omega$ . We study the behaviour of the first nonzero eigenvalue of this problem as the domain  $\Omega$  rotates about its own center by an angle  $\theta$  in the anticlockwise direction. We also study if there is any symmetry, monotonicity in the behaviour of the eigenvalue as a function of  $\theta$ , and try to find global maximisers and global minimisers of the eigenvalue with respect to  $\theta$ .



**ANALYSIS OF UPWIND DIFFERENCE SCHEME ON  
SHISHKIN-BAKHVALOV MESH FOR SINGULARLY  
PERTURBED TWO PARAMETER PARABOLIC  
PROBLEM WITH DISCONTINUOUS COEFFICIENTS**

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We consider a two-parameter singularly perturbed parabolic problem with a discontinuous convection coefficient and source term in two dimensions. The problem is discretized using an upwind difference scheme on a Shishkin-Bakhvalov mesh in space and the Crank-Nicolson scheme on a uniform mesh in time. At the point of discontinuity, a three-point formula is used. We prove that the numerical approximations are uniformly convergent (in the supremum norm) with respect to the singular perturbation parameters, when  $\epsilon \leq N^{-1}$  and  $N^{-1} \leq \epsilon$ . We achieve convergence, of first-order in space and second-order in time. Shishkin-Bakhvalov mesh provides first-order convergence; unlike the Shishkin mesh, where a logarithmic factor deteriorates the order of convergence. Some test examples are given to validate the theoretical order of convergence established for the numerical method.

## On the double phase elliptic problems with singular growth

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In this talk, we consider the boundary value problem  $-\Delta_p u_\lambda - \Delta_q u_\lambda = \lambda g(x) u_\lambda^{-\beta}$  in  $\Omega$ ,  $u_\lambda = 0$  on  $\partial\Omega$  with  $u_\lambda > 0$  in  $\Omega$ . We assume  $\Omega$  is a bounded open set in  $\mathbb{R}^N$  with smooth boundary,  $1 < p < q < \infty$ ,  $\beta \in [0, 1)$ ,  $g$  is a positive weight function and  $\lambda$  is a positive parameter. We derive an estimate for  $u_\lambda$  which describes its exact behavior when the parameter  $\lambda$  is large. In general, by invoking appropriate comparison principles, estimates of this kind can be used as a powerful tool in deducing the existence, non-existence and multiplicity of positive solutions of nonlinear elliptic boundary value problems. As an application of this estimate, we shall discuss a uniqueness result for a nonlinear elliptic boundary value problem with a singular nonlinearity.

# ON LORENTZ CONE SEMIPOSITIVITY OF MATRICES

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Given a proper cone  $K$  in the Euclidean space  $\mathbb{R}^n$ , a square matrix  $A$  is said to be  $K$ -semipositive if there exists an  $x \in K$  such that  $Ax \in \text{int}(K)$ , the topological interior of  $K$ . In this talk, we discuss algebraic and geometrical properties of  $\mathcal{L}_+^n$ -semipositive matrices, where the self-dual proper Lorentz cone  $\mathcal{L}_+^n$  is defined as  $\mathcal{L}_+^n = \{x \in \mathbb{R}^n : x_n \geq 0, \sum_{i=1}^{n-1} x_i^2 \leq x_n^2\}$ . We present a few necessary and other sufficient algebraic conditions for  $\mathcal{L}_+^n$ -semipositive matrices, and provide characterizations for diagonal and orthogonal  $\mathcal{L}_+^n$ -semipositive matrices. We next discuss the existence of basis of  $\mathbb{R}^{n \times n}$ , consisting of minimally  $\mathcal{L}_+^n$ -semipositive matrices.

Furthermore, given a square matrix  $A$  and a proper cone  $K$ , we present a few geometric properties of the semipositive cone  $\mathcal{K}_{A,K} = \{x \in K : Ax \in K\}$  and the cone of  $\mathcal{S}_{A,K} = \{x : Ax \in K\}$ , in terms of their extremals, and particular case is discussed taking  $K = \mathcal{L}_+^n$ .

# SHIFT AND OTHER OPERATORS ON THE HARDY-HILBERT SPACE

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The Hardy-Hilbert space, denoted by  $H^2$ , is the set of analytic functions whose power series have square summable coefficients. The study of Hardy-Hilbert spaces began in the early part of the twentieth century. There are various interesting classes of operators defined on the Hardy-Hilbert space. We begin our discussion with the unilateral shift operator because properties of this operator are useful in obtaining results on factorizations of analytic functions along with other aspects of analytic functions. At the same time, the Hardy-Hilbert context is also necessary to understand the structure of the invariant subspaces of the unilateral shift.

A study of the commutant of the unilateral shift operator leads us to another interesting class of operators on the Hardy-Hilbert space, namely the Toeplitz operators. The matrix of such an operator is constant along each diagonal. These are the most-studied and best-known operators on the Hardy Hilbert space.

Another class of operators, introduced in 1996, are the slant Toeplitz operators. The name “Slant Toeplitz” is motivated by the fact that the matrix of such an operator with respect to the standard orthonormal basis of  $H^2$  can be obtained by eliminating every alternate row of the matrix of the corresponding Toeplitz operator. These operators are in use in the theory of wavelet transforms, where they are known as sub-division operators.

Besides giving an overview of these operators on  $H^2$ , we will also try to see how the complexity of these operators increase manifold when they are defined on the space of functions analytic in the open unit ball of  $\mathbb{C}^n$ , denoted as  $\mathbb{B}_n = \{z \in \mathbb{C}^n : |z| < 1\}$ .

# On $\alpha$ -spectrum of Unicyclic Graphs and Some Estimations

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## Abstract

Let  $G = (V, E)$  be a simple connected graph with the adjacency matrix  $A(G)$  and let  $D(G)$  be the diagonal matrix of degrees of  $G$ . For any  $\alpha \in [0, 1]$ , the matrix  $A_\alpha(G) = \alpha A(G) + (1 - \alpha)D(G)$  is the  $A_\alpha$ -matrix of the graph  $G$ . The set of eigenvalues of  $A_\alpha$ -matrix is called the  $\alpha$ -spectrum of  $G$  and the largest eigenvalue of  $A_\alpha(G)$  is the  $\alpha$ -spectral radius of  $G$ . In this paper, we investigate some spectral properties of  $A_\alpha(U(n))$ , where  $U(n)$  refers to unicyclic graphs with  $n$  vertices. Also, we give some estimations on  $\alpha$ -spectral radius of Unicyclic graphs  $U(n)$ . We show that of all unicyclic graphs  $U(n, k)$  ( $3 \leq n, 0 \leq k \leq n - 3$ ) with  $n$  vertices and  $k$  pendant vertices, the maximal spectral radius is attained at  $u(n, k, r)$ , ( $3 \leq r \leq n, 0 \leq k \leq n - 3$ ), where  $u(n, k, r)$  is a unicyclic graph obtained by attaching  $k$  pendant paths of almost equal length to some point of a cycle  $C_r$ .

**Keywords:** Adjacency matrix;  $A_\alpha$ -matrix;  $\alpha$ -spectral radius; Unicyclic graph.

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# CESÁRO OPERATOR AND ITS GENERALIZATION ON BLOCH TYPE SPACES

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In this talk, we give a brief review of the classical Cesáro operator and Cesáro operators of order  $\alpha$  on various analytic function spaces such as Hardy, Bloch and *BMOA* spaces. We mainly focused on  $\mathcal{C}^\alpha$  operators defined by

$$\mathcal{C}^\alpha f(z) = \frac{(\alpha + 1)}{z^{\alpha+1}} \int_0^z f(w) \frac{(z-w)^\alpha}{(1-w)^{\alpha+1}} dw,$$

where  $\alpha > -1$  and  $f$  is analytic on the unit disc  $D$ . Boundedness properties of  $\mathcal{C}^\alpha$  between Bloch type spaces are discussed. We also find bound of the norm for  $\mathcal{C}^\alpha$  on  $B_0^a$  if  $a \in (1, \infty)$ . Further, we obtain  $\mathcal{C}^\alpha$  is bounded from Besov space into Bloch type space.

**Key words.** Cesáro operator, Bloch space, Besov space, Boundedness.

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